EE 303

SIGNALS & SYSTEMS

Question Bank #3

Due: Not collected (for your self-study)

1) For the continuous-time periodic signal

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right),$$

determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

2) Use the Fourier series analysis equation (3.39) to calculate the coefficients a_k for the continuous-time periodic signal

$$x(t) = \begin{cases} 1.5, & 0 \le t < 1 \\ -1.5, & 1 \le t < 2 \end{cases}$$

with fundamental frequency $\omega_0 = \pi$.

3) Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency ω_1 and Fourier coefficients a_k. Given that

$$x_2(t) = x_1(1 - t) + x_1(t - 1),$$

how is the fundamental frequency ω_2 of $x_2(t)$ related to ω_1? Also, find a relationship between the Fourier series coefficients b_k of $x_2(t)$ and the coefficients a_k. You may use the properties listed in Table 3.1.

4) Each of the two sequences $x_1[n]$ and $x_2[n]$ has a period $N = 4$, and the corresponding Fourier series coefficients are specified as

$$x_1[n] \longleftrightarrow a_k, \quad x_2[n] \longleftrightarrow b_k,$$

where

$$a_0 = a_3 = \frac{1}{2}a_1 = \frac{1}{2}a_2 = 1 \quad \text{and} \quad b_0 = b_1 = b_2 = b_3 = 1.$$

Using the multiplication property in Table 3.1, determine the Fourier series coefficients c_k for the signal $g[n] = x_1[n]x_2[n]$.

5) Consider three discrete-time systems S_1, S_2, and S_3 whose respective responses to a complex exponential input $e^{j\pi n/2}$ are specified as

$$S_1 : e^{j\pi n/2} \longrightarrow e^{j\pi n/2}u[n],$$
$$S_2 : e^{j\pi n/2} \longrightarrow e^{j3\pi n/2},$$
$$S_3 : e^{j\pi n/2} \longrightarrow 2e^{j5\pi n/2}.$$

For each system, determine whether the given information is sufficient to conclude that the system is definitely *not* LTI.

6) Let

$$x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2 - t, & 1 \le t \le 2 \end{cases}$$

be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k.

(a) Determine the value of a_0.

(b) Determine the Fourier series representation of $dx(t)/dt$.

(c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.

7) Consider the following three continuous-time signals with a fundamental period of $T = 1/2$:

$$x(t) = \cos(4\pi t),$$
$$y(t) = \sin(4\pi t),$$
$$z(t) = x(t)y(t).$$

(a) Determine the Fourier series coefficients of $x(t)$.

(b) Determine the Fourier series coefficients of $y(t)$.

(c) Use the results of parts (a) and (b), along with the multiplication property of the continuous-time Fourier series, to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.

(d) Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part (c).

8) Let

$$x[n] = \begin{cases} 1, & 0 \le n \le 7 \\ 0, & 8 \le n \le 9 \end{cases}$$

be a periodic signal with fundamental period $N = 10$ and Fourier series coefficients a_k. Also, let

$$g[n] = x[n] - x[n-1].$$

(a) Show that $g[n]$ has a fundamental period of 10.

(b) Determine the Fourier series coefficients of $g[n]$.

(c) Using the Fourier series coefficients of $g[n]$ and the First-Difference property in Table 3.2, determine a_k for $k \neq 0$.

9) Consider a causal continuous-time LTI system whose input $x(t)$ and output $y(t)$ are related by the following differential equation:

$$\frac{d}{dt}y(t) + 4y(t) = x(t).$$

Find the Fourier series representation of the output $y(t)$ for each of the following inputs:

(a) $x(t) = \cos 2\pi t$

(b) $x(t) = \sin 4\pi t + \cos(6\pi t + \pi/4)$

10) Consider a causal discrete-time LTI system whose input $x[n]$ and output $y[n]$ are related by the following difference equation:

$$y[n] - \frac{1}{4}y[n-1] = x[n]$$

Find the Fourier series representation of the output $y[n]$ for each of the following inputs:

(a) $x[n] = \sin(\frac{3\pi}{4}n)$

(b) $x[n] = \cos(\frac{\pi}{4}n) + 2\cos(\frac{\pi}{2}n)$

11) Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 1, & 0 \le n \le 2 \\ -1, & -2 \le n \le -1 \\ 0, & \text{otherwise} \end{cases}.$$

Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k],$$

determine the Fourier series coefficients of the output $y[n]$.

12) Suppose we are given the following information about a continuous-time periodic signal with period 3 and Fourier coefficients a_k:

1. $a_k = a_{k+2}$.
2. $a_k = a_{-k}$.
3. $\int_{-0.5}^{0.5} x(t)dt = 1$.
4. $\int_{1}^{2} x(t)dt = 2$.

Determine $x(t)$.

13) Suppose we are given the following information about a signal $x(t)$:

1. $x(t)$ is a real signal.
2. $x(t)$ is periodic with period $T = 6$ and has Fourier coefficients a_k.
3. $a_k = 0$ for $k = 0$ and $k > 2$.
4. $x(t) = -x(t-3)$.
5. $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 dt = \frac{1}{2}$.
6. a_1 is a positive real number.

Show that $x(t) = A\cos(Bt + C)$, and determine the values of the constants A, B, and C.

14) Consider the signal

$$x(t) = \cos 2\pi t.$$

Since $x(t)$ is periodic with a fundamental period of 1, it is also periodic with a period of N, where N is any positive integer. What are the Fourier series coefficients of $x(t)$ if we regard it as a periodic signal with period 3?

EE303 – Question Bank #3 (18/19 Fall)

(1) $x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$

for periodicity $x(t) = x(t+T)$ for all t

$$x(t+T) = 2 + \cos\left(\frac{2\pi}{3}t + \underbrace{\frac{2\pi}{3}T}_{=2\pi k}\right) + 4\sin\left(\frac{5\pi}{3}t + \underbrace{\frac{5\pi}{3}T}_{=2\pi k}\right)$$

$\frac{2\pi}{3}T = 2\pi k \Rightarrow T_1 = \frac{k}{3}$ $\quad$ $\frac{5\pi}{3}T = 2\pi k \Rightarrow T_2 = \frac{6k}{5}$

$$T_0 = LCM\left(\frac{1}{3}, \frac{6}{5}\right) = 6 \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{6} = \pi/3$$

$$x(t) = 2 + \frac{e^{j\frac{2\pi}{3}t} + e^{-j\frac{2\pi}{3}t}}{2} + 4\,\frac{e^{j\frac{5\pi}{3}t} + e^{-j\frac{5\pi}{3}t}}{2j}$$

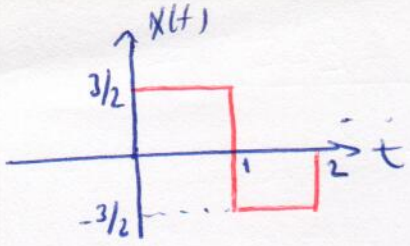
$$= 2 + \frac{1}{2}e^{j2\omega_0 t} + \frac{1}{2}e^{-j2\omega_0 t} + \frac{2}{j}e^{j5\omega_0 t} - \frac{2}{j}e^{-j5\omega_0 t}$$

$\Rightarrow$ here, $a_0 = 2$

$a_2 = \frac{1}{2}$; $a_{-2} = \frac{1}{2}$

$a_5 = \frac{2}{j}$; $a_{-5} = -\frac{2}{j}$ & other a_k's are all zero.

(2)



$\omega_0 = \pi$

$$a_k = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{2}\int_0^2 x(t)\, e^{-jk\pi t}\, dt$$

$$= \frac{1}{2}\left(\frac{3}{2}\int_0^1 e^{-jk\pi t}\, dt - \frac{3}{2}\int_1^2 e^{-jk\pi t}\, dt\right)$$

$$= \frac{3}{4}\left\{\frac{e^{-jk\pi t}}{-jk\pi}\Big|_0^1 - \frac{e^{-jk\pi t}}{-jk\pi}\Big|_1^2\right\}$$

$$= \frac{3}{4}\left\{\frac{e^{-jk\pi} - 1}{-jk\pi} - \frac{e^{-jk2\pi} - e^{-jk\pi}}{-jk\pi}\right\}$$

$$= \frac{-3}{4jk\pi}\left(2\underbrace{e^{-jk\pi}}_{(-1)^k} - 1 - \underbrace{e^{-jk2\pi}}_{1}\right)$$

$$= \frac{3}{4jk\pi}\left(2 - 2(-1)^k\right) = \frac{3}{j2k\pi}\left(1 - (-1)^k\right) \leftarrow$$

(3) $x_1(t) \longleftrightarrow a_k \quad (\omega_1)$

$x_2(t) = x_1(1-t) + x_1(t-1)$ $\qquad$ $x_2(t) \longleftrightarrow b_k \quad (\omega_2)$

As $x_1(t)$ has a fundamental freq. of ω_1, then $x_2(t)$ has also some fund. freq. due to linearity $\Rightarrow$ $\boxed{\omega_2 = \omega_1}$

$x_1(t) \longleftrightarrow a_k \Rightarrow x_1(t-1) \longleftrightarrow e^{-j\omega_1} a_k$

$x_1(1-t) \longleftrightarrow -e^{-j\omega_1} a_{-k}$

by using linearity prop.

$\Rightarrow$ $\boxed{b_k = e^{-j\omega_1} a_k - e^{-j\omega_1} a_{-k}}$

(4) $x_1[n] \longleftrightarrow a_k$, $x_2[n] \longleftrightarrow b_k$ $\qquad$ $N=4$

$a_0 = a_3 = 1$ $\qquad$ $b_0 = b_1 = b_2 = b_3 = 1$

$a_1 = a_2 = -1$

$g[n] = x_1[n] \cdot x_2[n] \longleftrightarrow c_k = ?$

$g[n] = x_1[n] \cdot x_2[n] \Rightarrow c_k = \sum_{\ell = \langle N \rangle} a_\ell b_{k-\ell} = \sum_{\ell=0}^{3} a_\ell b_{k-\ell}$

$c_k = a_0 b_k + a_1 b_{k-1} + a_2 b_{k-2} + a_3 b_{k-3}$

$c_k = b_k - b_{k-1} - b_{k-2} + b_{k-3}$

$k<0 \Rightarrow c_k = 0$

$k=0 \rightarrow c_0 = b_0 - b_{-1} - b_{-2} + b_{-3} = 1$

$k=1 \rightarrow c_1 = 1 - 1 = 0$

$k=2 \rightarrow c_2 = 1 - 1 - 1 = -1$

$k=3 \rightarrow c_3 = 1 - 1 - 1 + 1 = 0$

$k=4 \Rightarrow c_4 = 0 - 1 - 1 + 1 = -1$

$k=5 \quad c_5 = 0 - 0 - 1 + 1 = 0$

$c_6 = 0 - 0 - 0 + 1 = 1$

$k \geq 7 \Rightarrow c_k = 0$

5)

$S_1[n]: e^{j\pi n/2} \longrightarrow e^{j\pi n/2}\, u(n)$

$S_2[n]: e^{j\pi n/2} \longrightarrow e^{j3\pi n/2}$

$S_3[n]: e^{j\pi n/2} \longrightarrow 2e^{j5\pi n/2}$

For LTI systems, complex exponentials are eigenvalues of them. Hence

$e^{st} \longrightarrow A \cdot e^{st}$ must be satisfied.

A: real or complex constant.

i) $e^{j\pi n/2} \longleftrightarrow e^{j\pi n/2} \cdot u(n) \Longrightarrow$ Definitely not LTI

ii) $e^{j\pi n/2} \longleftrightarrow e^{j3\pi n/2} = e^{j(\pi+\frac{\pi}{2})n/2} = e^{j\pi n/2} \cdot \underbrace{e^{j\pi/2\, n/2}}_{e^{j\frac{\pi}{4}n}:\text{not const.}}$

$\Rightarrow$ Definitely not LTI

iii) $e^{j\pi n/2} \longleftrightarrow 2 \cdot e^{j5\pi n/2} = 2 \cdot \underbrace{e^{j2\pi n}}_{=1} \cdot e^{j\pi n/2} = 2 \cdot e^{j\pi n/2}$

$\Rightarrow$ hence this can be an LTI system.

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6) $x(t) = \begin{cases} t & ; \ 0 \le t \le 1 \\ 2-t & ; \ 1 \le t \le 2 \end{cases}$ $\quad \underline{T=2} \Rightarrow a_k$

$\downarrow \ w_0 = \frac{2\pi}{T} = \pi/2$

$$a_k = \frac{1}{2}\int_0^2 x(t)\, e^{-jkw_0 t}\, dt = \frac{1}{2}\int_0^2 x(t)\, e^{-jk\pi t}\, dt$$

a) $$a_0 = a_k\Big|_{k=0} = \frac{1}{2}\int_0^2 x(t)\, dt = \frac{1}{2}\left\{\int_0^1 t\, dt + \int_1^2 (2-t)\, dt\right\} = \underline{1}$$

b) $y(t) = \frac{d}{dt}x(t) \longleftrightarrow b_k$ $\qquad y(t) = \sum_{k=-\infty}^{\infty} b_k e^{+jkw_1 t}$ $\Rightarrow \underline{w_1 = w_0}$

$\underline{b_k = jk\frac{2\pi}{T} a_k}$ $\qquad y(t) = j\frac{2\pi}{T}\sum_k k \cdot e^{jk2\pi t} \cdot a_k = \frac{d}{dt}x(t)$

c) $\Rightarrow$ $\boxed{a_k = \frac{1}{2\pi}\frac{1}{jk} b_k}$

(7)

$$\left.\begin{aligned} x(t) &= \cos(4\pi t) \\ y(t) &= \sin(4\pi t) \\ z(t) &= x(t)\cdot y(t) \end{aligned}\right\} \quad T = 1/2 \Rightarrow \omega_0 = \frac{2\pi}{T} = 4\pi$$

(a) $x(t) = \cos 4\pi t = \frac{1}{2} e^{j4\pi t} + \frac{1}{2} e^{-j4\pi t} = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$

$x(t) \longleftrightarrow a_k$

$$a_k = \begin{cases} 1/2 \; ; \; k = \mp 1 \\ 0 \; ; \; \text{otherwise} \end{cases}$$

(b) $y(t) \longleftrightarrow b_k$

$y(t) = \sin 4\pi t = \sin \omega_0 t = \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t} \Rightarrow b_k = \begin{cases} \frac{1}{2j} \; ; \; k=1 \\ -\frac{1}{2j} \; ; \; k=-1 \\ 0 \; ; \; \text{oth.} \end{cases}$

(c) $z(t) \longleftrightarrow c_k$

by using multiplication property

$$\text{if } z(t) = x(t).y(t) \longleftrightarrow c_k = \sum_{\ell} a_\ell b_{k-\ell} = \sum_{\ell=0}^{1} a_\ell b_{k-\ell}$$

$$c_k = \frac{1}{2}\left(b_{k-1} + b_{k+1}\right) \; ; \; k=0 \; ; \; c_0 = \frac{1}{2}\left(\frac{1}{2j} - \frac{1}{2j}\right) = 0$$

$k=1 \; ; \; c_1 = 0$

$k=2 \; ; \; c_2 = \frac{1}{2}(b_1 + b_3) = \frac{1}{4j}$

$k=-2 \; ; \; c_{-2} = \frac{1}{2}(b_{-3} + b_{-1}) = -\frac{1}{4j}$

$$c_k = \begin{cases} \frac{1}{4j} \; ; \; k=2 \\ -\frac{1}{4j} \; ; \; k=-2 \\ 0 \; ; \; \text{oth.} \end{cases}$$

(d) $z(t) = x(t).y(t) = \cos(4\pi t)\sin(4\pi t) = \frac{\sin(8\pi t) - \sin(0)}{2} = \frac{1}{2}\sin 8\pi t$

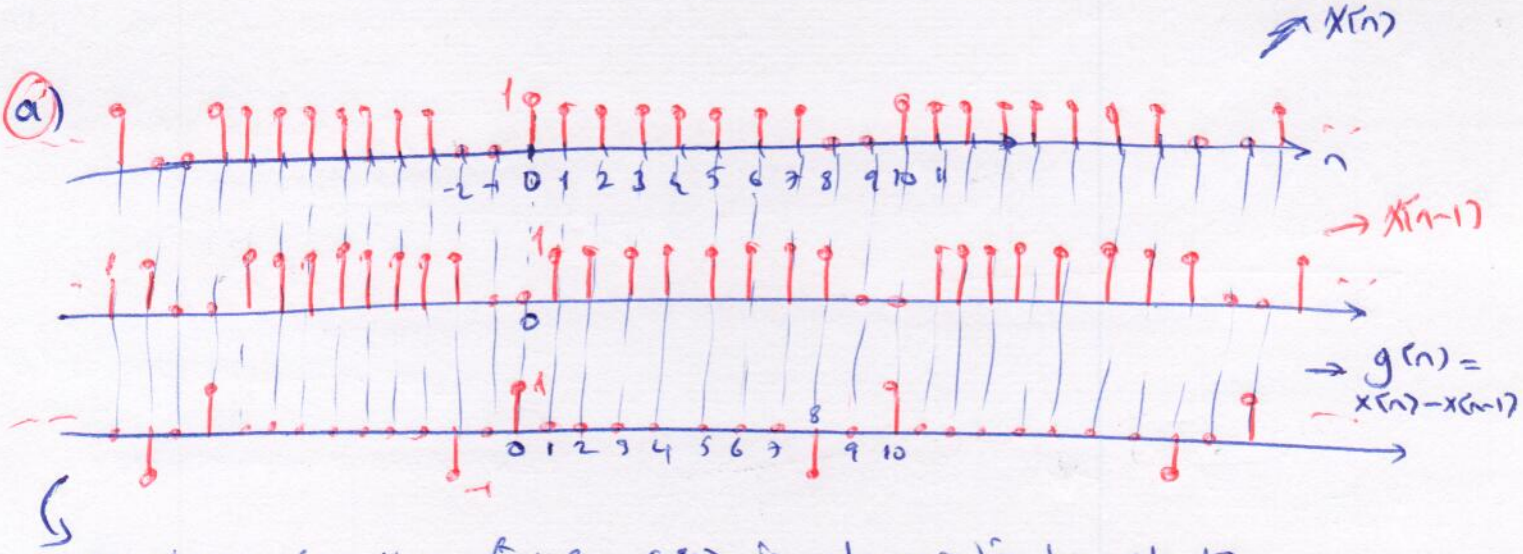
$$= \frac{1}{2}\left\{\frac{e^{j8\pi t} - e^{-j8\pi t}}{2j}\right\} = \frac{1}{4j}\left\{e^{j2\omega_0 t} - e^{-j2\omega_0 t}\right\} \Rightarrow \begin{aligned} c_2 &= \frac{1}{4j} \\ c_{-2} &= -\frac{1}{4j} \end{aligned}$$

(8)

$$x[n] = \begin{cases} 1 ; & 0 \le n \le 7 \\ 0 ; & 8 \le n \le 9 \end{cases}, \quad \underline{N=10}, \quad x[n] \longleftrightarrow a_k$$

$$g[n] = x[n] - x[n-1]$$

a)



as shown from this figure $g[n]$ is also periodic by $\underline{N=10}$

b) $x[n] \longleftrightarrow a_k$

$g[n] \longleftrightarrow b_k$ $\quad g[n] = x[n] - x[n-1] \longleftrightarrow b_k = a_k - e^{-jw_0} a_k$

$$= (1 - e^{-jw_0}) a_k$$

$$\underline{b_k = (1 - e^{-j\frac{\pi}{5}}) a_k}$$

c)

$$g[n] = \frac{1}{N} \sum_k b_k e^{jk\frac{2\pi}{N}n} = \delta[n] - \delta[n-8] = \frac{1}{10} \sum_k b_k e^{jk\frac{\pi}{5}n}$$

we know $g[n] = \begin{cases} 1 ; & n=0 \\ -1 ; & n=8 \\ 0 ; & \text{o.w.} \end{cases}$

$$g[0] = 1 = \frac{1}{10} \sum_k b_k$$

$$g[8] = -1 = \frac{1}{10} \sum_k b_k e^{-jk\frac{8\pi}{5}}$$

$$b_k = \frac{1}{N} \sum_n g[n] e^{-j\frac{2\pi}{N}kn} = \frac{1}{10} \left(1 - e^{-j\frac{2\pi}{10}8k}\right)$$

($n=0$, $n=8$)

$$a_k = \frac{b_k}{1 - e^{-j\frac{2\pi}{10}}} = \frac{1}{10} \left(\frac{1 - e^{-j\frac{2\pi}{10}8k}}{1 - e^{-j\frac{\pi}{5}}} \right)$$

(9) Causal, LTI, CT system $\frac{d}{dt}y(t) + 4y(t) = x(t)$

$x(t) \longleftrightarrow a_k \qquad y(t) \longleftrightarrow b_k$

$\frac{d}{dt}y(t) \longleftrightarrow jk\frac{2\pi}{T} b_k$

by using linearity property;

$$jk\frac{2\pi}{T} b_k + 4b_k = a_k \Rightarrow \boxed{b_k = -\frac{1}{4 + jk\frac{2\pi}{T}} a_k}$$

(a) $x(t) = \cos 2\pi t \longleftrightarrow a_k = ?$

$= \frac{1}{2} e^{j2\pi t} + \frac{1}{2} e^{-j2\pi t} \Rightarrow T = 1, \; \omega_0 = 2\pi/T = 2\pi \Rightarrow a_k = \begin{cases} 1/2 & ; k = 1, -1 \\ 0 & ; o.w. \end{cases}$

$$b_k = \begin{cases} \frac{-1/2}{4 + jk2\pi} & ; k = 1, -1 \\ 0 & ; o.w. \end{cases}$$

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk2\pi t}$$

$$= \frac{-1/2}{4 + j2\pi} \cdot e^{-j2\pi t} + \frac{-1/2}{4 - j2\pi} e^{j2\pi t}$$

(b) $x(t) = \sin 4\pi t + \cos(6\pi t + \pi/4)$

$$= \frac{e^{j4\pi t} - e^{-j4\pi t}}{2j} + \frac{e^{j6\pi t} \cdot e^{j\pi/4} + e^{-j6\pi t} \cdot e^{-j\pi/4}}{2} \qquad \omega_0 = 2\pi, \; T = 1$$

$$x(t) = \frac{1}{2j} e^{j2\omega_0 t} - \frac{1}{2j} e^{-j2\omega_0 t} + \frac{1}{2} e^{j\pi/4} \cdot e^{j3\omega_0 t} + \frac{1}{2} e^{-j\pi/4} e^{-j3\omega_0 t}$$

$$a_k = \begin{cases} \frac{1}{2j} & ; k = 2 \\ -1/2j & ; k = -2 \\ \frac{1}{2} \cdot e^{j\pi/4} & ; k = 3 \\ -1/2\, e^{j\pi/4} & ; k = -3 \\ 0 & ; o.w. \end{cases} \Rightarrow b_k = \frac{-1/2}{4 + jk2\pi} \times \begin{cases} \frac{1}{j} & ; k = 2 \\ -\frac{1}{j} & ; k = -2 \\ e^{j\pi/4} & ; k = 3 \\ -e^{j\pi/4} & ; k = -3 \\ 0 & ; o.w. \end{cases}$$

$$y(t) = \sum_{k=2,-2,3,-3} b_k \cdot e^{jk\omega_0 t}$$

(10) causal, LTI, DT system $y[n] - \frac{1}{4}y[n-1] = x[n]$

$x[n] \longleftrightarrow a_k$

$y[n] \longleftrightarrow b_k \qquad y[n-1] \longleftrightarrow e^{-j\frac{2\pi}{N}k} b_k$

$$b_k - \frac{1}{4}e^{-j\omega_0} b_k = a_k \Rightarrow \boxed{\left(1 - \frac{1}{4}e^{-j\omega_0}\right) b_k = a_k} \quad ; \omega_0 = \frac{2\pi}{N}$$

(a) $x[n] = \sin\left(\frac{3\pi}{4}n\right) = \frac{e^{j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n}}{2} = \frac{1}{2}e^{j3\omega_0 n} + \frac{1}{2}e^{-j3\omega_0 n}$; $\omega_0 = \frac{\pi}{4}$ ($N=8$)

$$a_k = \begin{cases} \frac{1}{2} & ; k = \mp 3 \\ 0 & ; \text{oth.} \end{cases}$$

$$b_k = \frac{1}{1 - \frac{1}{4}e^{-j\frac{2\pi}{8}}} \cdot a_k \Rightarrow b_k = \begin{cases} \frac{4/2}{4 - e^{j\pi/4}} & ; k = \mp 3 \\ 0 & ; \text{oth.} \end{cases}$$

$$y[n] = \sum_{k=-3,3} b_k e^{j\frac{2\pi}{N}kn} = \frac{2}{4 - e^{j\pi/4}}\left(e^{-j\frac{3\pi}{8}n} + e^{j\frac{3\pi}{8}n}\right) \leftarrow$$

(b) $x[n] = \cos\left(\frac{\pi}{4}n\right) + 2\cos\left(\frac{\pi}{2}n\right) = \frac{1}{2}e^{j\pi/4 n} + \frac{1}{2}e^{-j\frac{\pi}{4}n} + e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}$

$\omega_0 = \frac{\pi}{4}$ ($\gcd(\frac{\pi}{4}, \frac{\pi}{2})$

$x[n] = \frac{1}{2}e^{j\omega_0 n} + \frac{1}{2}e^{-j\omega_0 n} + e^{j2\omega_0 n} + e^{-j2\omega_0 n}$

$$a_k = \begin{cases} \frac{1}{2} & ; k = 1, -1 \\ 1 & ; k = 2, -2 \\ 0 & ; \text{oth.} \end{cases} \Rightarrow b_k = \frac{4}{4 - e^{j\pi/4}} a_k \leftarrow$$

(11)

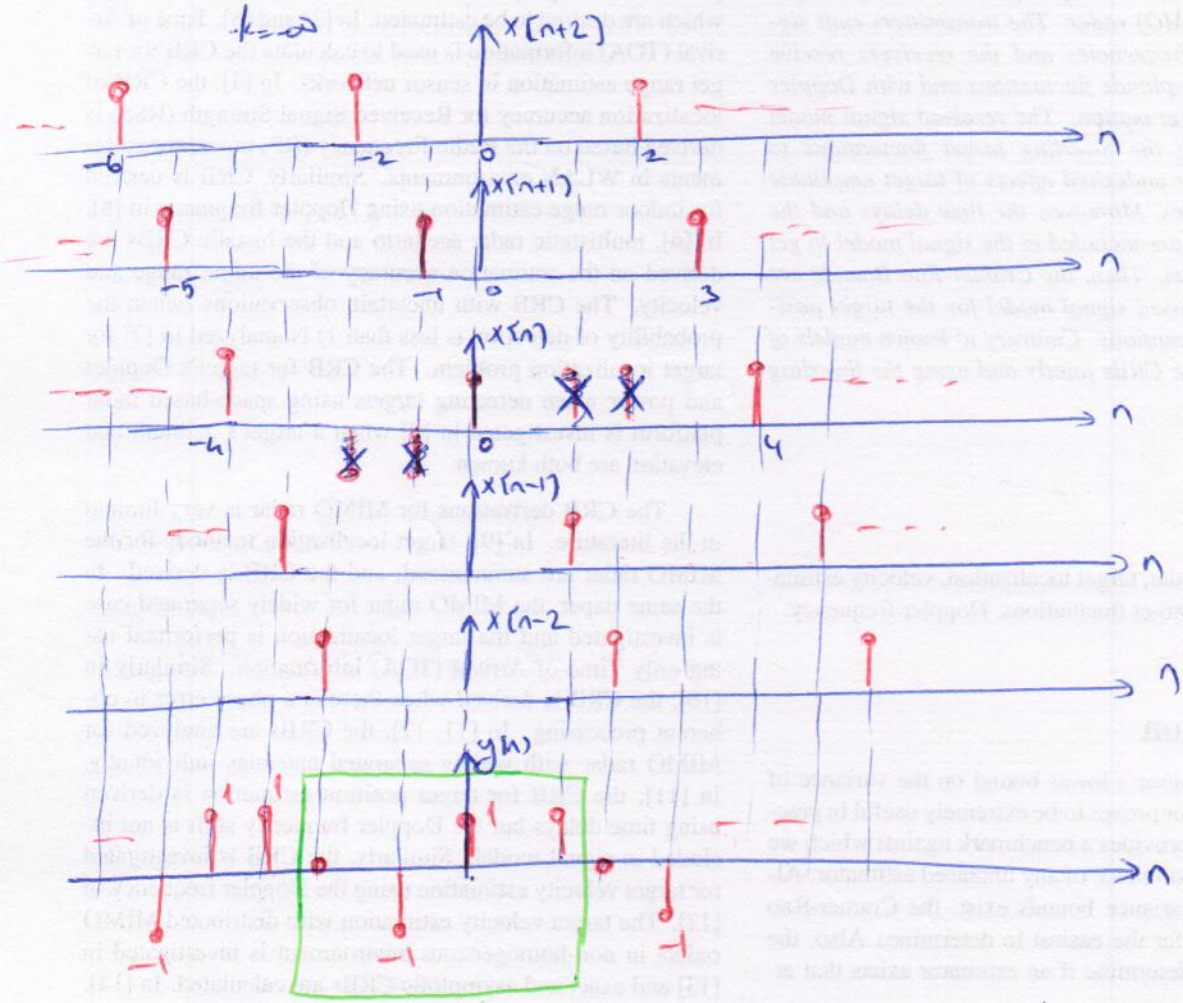
$$h[n] = \begin{cases} 1 & ; \ 0 \le n \le 2 \\ -1 & ; \ -2 \le n \le -1 \\ 0 & ; \ \text{oth.} \end{cases} \quad , \quad x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$$

$x[n] \longleftrightarrow a_k \Rightarrow y[n] \longleftrightarrow b_k = ?$

$y[n] = x[n] * h[n]$



$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] = -x[n+2] - x[n+1] + x[n] + x[n-1] + x[n-2]$$



> it is again periodic by $N=4$

$$b_k = \frac{1}{N} \sum_{n=<N>} y[n]\, e^{-j\frac{2\pi}{N}kn} = \frac{1}{4} \sum_{n=0}^{3} y[n]\, e^{-j\frac{2\pi}{4}kn}$$

$$= \frac{1}{4}\left\{ e^{-j\frac{\pi}{2}n} + e^{-j3\left(\frac{\pi}{2}\right)n} + 1 \right\} = \frac{1}{4}\left\{ 1 + e^{-j\frac{\pi}{2}n} - e^{-j\frac{3\pi}{2}n} \right\}$$

$$b_k = \frac{1}{4}\left\{ 1 + e^{-j\pi n}\left(e^{j\pi/2 n} - e^{-j\pi/2 n} \right) \right\} = \frac{1}{4}\left(1 + 2j \sin\left(\frac{\pi}{2}n\right) \cdot e^{-j\pi/4 n} \right)$$

(12) CT periodic signal with $T=3$ $\quad x(t) \longleftrightarrow a_k$

$$a_k = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$a_k = \frac{1}{3}\int_0^3 x(t)\, e^{-jk\frac{2\pi}{3}t}\, dt \quad (*)$$

for $x(t) \Longrightarrow x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$; $\omega_0 = \frac{2\pi}{3}$

from (2) we have $a_k = a_{-k} \Longrightarrow x(t) = \sum_{k=0}^{\infty} a_k \left(e^{jk\omega_0 t} + e^{-jk\omega_0 t}\right)$

$$= \sum_{k=0}^{\infty} a_k \cdot 2\cos(k\omega_0 t)$$

$$x(t) = a_0 + 2\left\{a_1\cos\omega_0 t + a_2\cos(2\omega_0 t) + a_3\cos(3\omega_0 t) + a_4\cos(4\omega_0 t) + \cdots\right\}$$

$k \to \infty$

we know (1) $a_k = a_{k+2} \Longrightarrow a_0 = a_2 = a_4 = a_6 = \cdots$

$a_1 = a_3 = a_5 = \cdots$

$$x(t) = a_0 + 2\left\{a_1\cos\omega_0 t + a_0\cos 2\omega_0 t + a_1\cos 3\omega_0 t + a_0\cos 4\omega_0 t + \cdots\right\}$$

$$= a_0 + 2\left\{a_1\cos\frac{2\pi}{3}t + a_0\cos\frac{4\pi}{3}t + a_1\cos 2\pi t + a_0\cos\left(\frac{8\pi}{3}t\right) + \cdots\right.$$

$$a_0 = \frac{1}{3}\int_{\langle T\rangle} x(t)\, dt = \frac{1}{3}\left(\int_0^3 x(t)\, dt\right) = \frac{1}{3}\left(\int_{-0.5}^{0.5} x(t)\, dt + \int\right.$$

(12) (1) $x(t)$ is a real signal ← $x(t) \longleftrightarrow a_k$

(2) $x(t) = \sum_k a_k e^{jk\omega_0 t}$ $\quad x(t) = x^*(t) = \sum_k a_k^* e^{-jk\omega_0 t}$

(3) $T = 6$

$$x(t) = \sum_k a_k e^{jk\frac{2\pi}{6}t} = \sum_k a_k^* e^{-jk\frac{2\pi}{6}t}$$

$$a_k = \begin{cases} 0 & ; k=0 \\ a_1 & ; k=1 \\ a_2 & ; k=2 \\ 0 & ; k>0 \\ ? & ; k<0 \end{cases}$$

$$\sum_k a_k e^{jk\frac{2\pi}{6}t} = \sum_m a_{-m}^* e^{jm\frac{2\pi}{6}t}$$

$$\boxed{a_k = a_{-k}^*}$$

$$a_k = \begin{cases} a_1^* & ; k=-1 \\ a_2^* & ; k=-2 \\ 0 & ; k<-2 \end{cases}$$

⇒ so a_k has only 4 value!

(4) $x(t) = -x(t-3)$

(14)

$x(t) = \cos 2\pi t \Rightarrow T = 1 \quad \omega_0 = 2\pi$

also for $T = N$ (N is integer)

for $T=1$

$x(t) = \cos 2\pi t \longleftrightarrow a_k$

$= \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t} \longleftrightarrow a_k = \begin{cases} \frac{1}{2} & ; k = 1, -1 \\ 0 & ; \text{o.w.} \end{cases}$

for $T=3$

$b_k = ?$

$$b_k = \frac{1}{T} \int_{\langle T \rangle} x(t)\, e^{-j\omega_0 t} dt = \frac{1}{3} \int_{\langle 3 \rangle} x(t) \cdot e^{-jk\frac{2\pi}{3}t} dt$$

$$= \frac{1}{3} \int_{\langle 3 \rangle} \left(\frac{1}{2} e^{jk2\pi t} + \frac{1}{2} e^{-jk2\pi t} \right) e^{-j\frac{2\pi}{3}t} dt$$

$$b_k = \frac{1}{6} \int_{\langle 3 \rangle} e^{j(2\pi k - \frac{2\pi}{3})t} dt + \frac{1}{6} \int_{\langle 3 \rangle} e^{-j(2\pi k + \frac{2\pi}{3})t} dt$$

→ add k !!

$$= \frac{1}{6} \left\{ \int_0^3 \left(e^{j\frac{4\pi}{3}t} + e^{-j\frac{8\pi}{3}t} \right) dt \right\}$$

$$= \frac{1}{6} \left\{ \frac{e^{j\frac{4\pi}{3}t}}{j\frac{4\pi}{3}} + \frac{e^{-j\frac{8\pi}{3}t}}{-j\frac{8\pi}{3}} \Bigg|_0^3 \right\}$$

$$= \frac{1}{6} \frac{3}{j4\pi} \left\{ e^{j\frac{4\pi}{3}t} + \frac{1}{2} e^{-j\frac{8\pi}{3}t} \Bigg|_0^3 \right\} = \frac{1}{j8\pi} \left(e^{j4\pi} + \frac{1}{2} e^{-j8\pi} - 1 - \frac{1}{2} \right)$$