EE 303

SIGNALS & SYSTEMS

Question Bank #2

Due: Not collected (for your self-study)

1) Let

$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3] \quad \text{and} \quad h[n] = 2\delta[n+1] + 2\delta[n-1].$$

Compute and plot each of the following convolutions:

(a) $y_1[n] = x[n] * h[n]$ **(b)** $y_2[n] = x[n+2] * h[n]$

(c) $y_3[n] = x[n] * h[n+2]$

2) Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \begin{cases} 1, & 3 \le n \le 8 \\ 0, & \text{otherwise} \end{cases},$$

$$h[n] = \begin{cases} 1, & 4 \le n \le 15 \\ 0, & \text{otherwise} \end{cases}.$$

3) Let

$$x[n] = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & \text{elsewhere} \end{cases} \quad \text{and} \quad h[n] = \begin{cases} 1, & 0 \le n \le N \\ 0, & \text{elsewhere} \end{cases},$$

where $N \le 9$ is an integer. Determine the value of N, given that $y[n] = x[n] * h[n]$ and

$$y[4] = 5, \quad y[14] = 0.$$

4) A linear system S has the relationship

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k],$$

between its input $x[n]$ and its output $y[n]$, where $g[n] = u[n] - u[n-4]$.

(a) Determine $y[n]$ when $x[n] = \delta[n-1]$.

(b) Determine $y[n]$ when $x[n] = \delta[n-2]$.

(c) Is S LTI?

(d) Determine $y[n]$ when $x[n] = u[n]$.

5) Determine and sketch the convolution of the following two signals:

$$x(t) = \begin{cases} t+1, & 0 \le t \le 1 \\ 2-t, & 1 < t \le 2 \\ 0, & \text{elsewhere} \end{cases},$$

$$h(t) = \delta(t+2) + 2\delta(t+1).$$

6) Let

$$x(t) = u(t-3) - u(t-5) \quad \text{and} \quad h(t) = e^{-3t}u(t).$$

(a) Compute $y(t) = x(t) * h(t)$.
(b) Compute $g(t) = (dx(t)/dt) * h(t)$.
(c) How is $g(t)$ related to $y(t)$?

7) Let $h(t)$ be the triangular pulse shown in Figure P2.23(a), and let $x(t)$ be the impulse train depicted in Figure P2.23(b). That is,

$$x(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT).$$

Determine and sketch $y(t) = x(t) * h(t)$ for the following values of T:
(a) $T = 4$ **(b)** $T = 2$ **(c)** $T = 3/2$ **(d)** $T = 1$

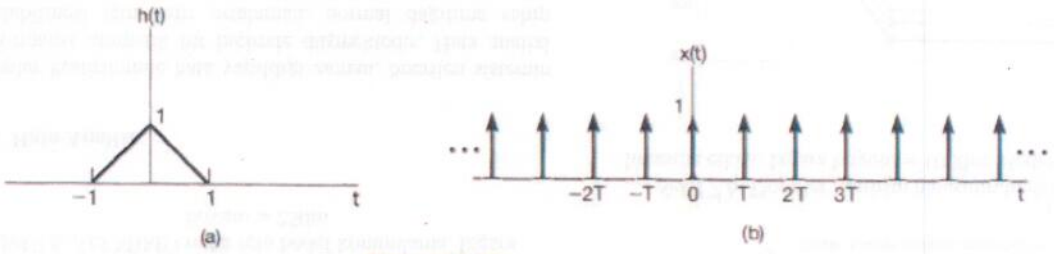


Figure P2.23

8) Consider the cascade interconnection of three causal LTI systems, illustrated in Figure P2.24(a). The impulse response $h_2[n]$ is

$$h_2[n] = u[n] - u[n-2],$$

and the overall impulse response is as shown in Figure P2.24(b).

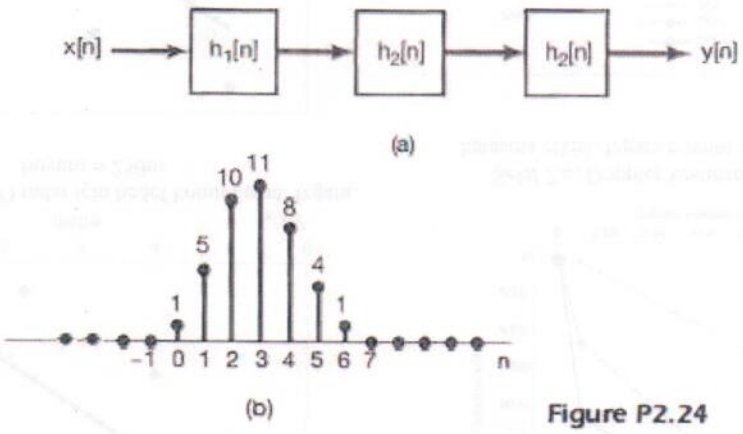


Figure P2.24

(a) Find the impulse response $h_1[n]$.
(b) Find the response of the overall system to the input

$$x[n] = \delta[n] - \delta[n-1].$$

9) The following are the impulse responses of discrete-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

(a) $h[n] = (\frac{1}{5})^n u[n]$

(b) $h[n] = (0.8)^n u[n+2]$

(c) $h[n] = (\frac{1}{2})^n u[-n]$

(d) $h[n] = (5)^n u[3-n]$

(e) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[n-1]$

(f) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[1-n]$

(g) $h[n] = n(\frac{1}{3})^n u[n-1]$

10) The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

(a) $h(t) = e^{-4t}u(t-2)$

(b) $h(t) = e^{-6t}u(3-t)$

(c) $h(t) = e^{-2t}u(t+50)$

(d) $h(t) = e^{2t}u(-1-t)$

(e) $h(t) = e^{-6|t|}$

(f) $h(t) = te^{-t}u(t)$

(g) $h(t) = (2e^{-t} - e^{(t-100)/100})u(t)$

11) Consider the LTI system initially at rest and described by the difference equation

$$y[n] + 2y[n-1] = x[n] + 2x[n-2].$$

Find the response of this system to the input depicted in Figure P2.31 by solving the difference equation recursively.

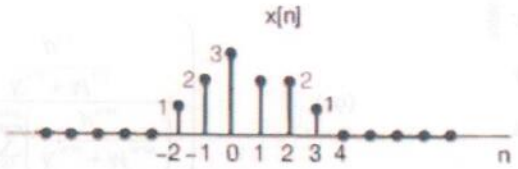


Figure P2.31

12) Consider the cascade of two systems shown in Figure P2.50. The first system, A, is known to be LTI. The second system, B, is known to be the inverse of system A. Let $y_1(t)$ denote the response of system A to $x_1(t)$, and let $y_2(t)$ denote the response of system A to $x_2(t)$.

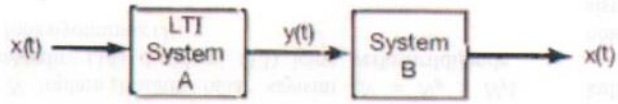


Figure P2.50

(a) What is the response of system B to the input $ay_1(t) + by_2(t)$, where a and b are constants?

(b) What is the response of system B to the input $y_1(t-\tau)$?

EE303 – Question Bank #2 (18/19 Fall)

① $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$, $h[n] = 2\delta[n+1] + 2\delta[n-1]$

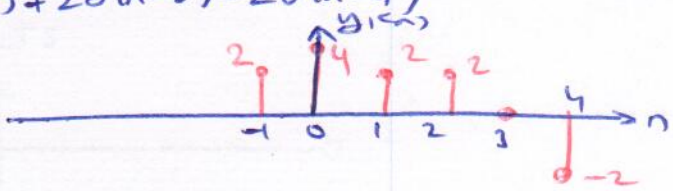
a) $y_1[n] = x[n] * h[n]$

$= (\delta[n] + 2\delta[n-1] - \delta[n-3]) * h[n]$

$= h[n] + 2h[n-1] - h[n-3]$

$= 2\delta[n+1] + 2\delta[n-1] + 4\delta[n] + 4\delta[n-2] - 2\delta[n-2] - 2\delta[n-4]$

$= 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$



b) $y_2[n] = x[n+2] * h[n]$

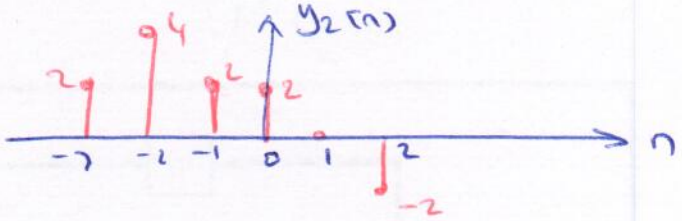
$= (\delta[n+2] + 2\delta[n+1] - \delta[n-1]) * h[n]$

$= h[n+2] + 2h[n+1] - h[n-1]$

$= 2\delta[n+3] + 2\delta[n+1] + 4\delta[n+2] + 4\delta[n] - 2\delta[n] - 2\delta[n-2]$

$= 2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1] + 2\delta[n] - 2\delta[n-2]$

$= y_1[n+2]$



c) $y_3[n] = x[n] * h[n+2] = x[n] * \{2\delta[n+3] + 2\delta[n+1]\}$

$= 2x[n+3] + 2x[n+1]$

$y_3[n] = 2\delta[n+3] + 2\delta[n+1] + 4\delta[n+2] + 4\delta[n] - 2\delta[n] - 2\delta[n-2]$

$= 2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1] + 2\delta[n] - 2\delta[n-2]$

$= y_1[n+2]$

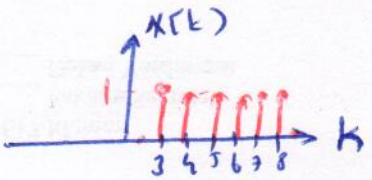
$= y_2[n]$

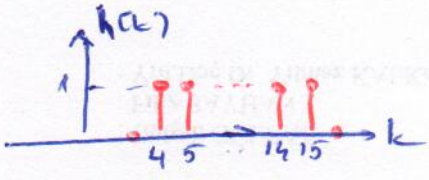
$x[n] * h[n+2] = x[n+2] * h[n]$

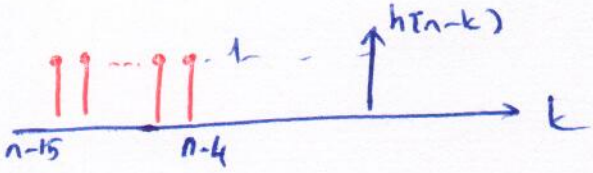
(2)

$$x[n] = \begin{cases} 1 ; & 3 \le n \le 8 \\ 0 ; & \text{oth.} \end{cases} \qquad h[n] = \begin{cases} 1 ; & 4 \le n \le 15 \\ 0 ; & \text{oth.} \end{cases} \qquad y[n] = ?$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \sum_{k=3}^{8} h[n-k] \quad ?$$







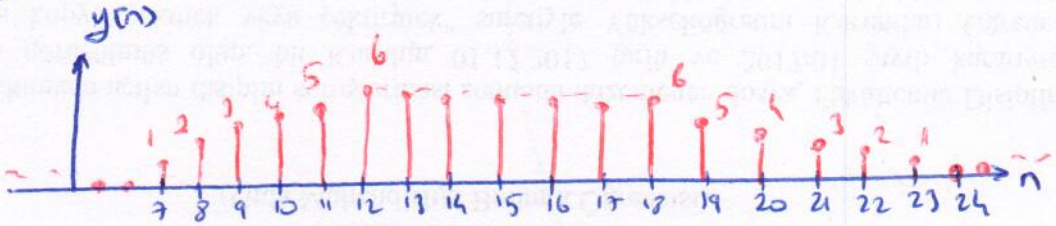
1) $n-4 < 3 \Rightarrow y[n] = 0 \Rightarrow n < 7 \; ; \; y[n] = 0$

2) $n-4 \ge 3 \;\&\; n-4 \le 8 \Rightarrow 7 \le n \le 12 \Rightarrow y[n] = \sum_{k=3}^{n-4} 1 = n-6$

3) $n-4 > 8 \;\&\; n-15 \le 3 \quad 12 < n \le 18 \Rightarrow y[n] = \sum_{k=3}^{8} 1 = 6$

4) $n-15 > 3 \;\&\; n-15 \le 8 \quad 18 < n \le 23 \Rightarrow y[n] = \sum_{k=n-15}^{8} 1 = n-6$

5) $n > 23 \; ; \; y[n] = 0 \; ;$



3) $x[n] = \begin{cases} 1 ; & 0 \le n \le 9 \\ 0 ; & \text{oth.} \end{cases}$ $h[n] = \begin{cases} 1 ; & 0 \le n \le N \\ 0 ; & \text{oth.} \end{cases}$ $N \le 9$ integer.

$y[n] = x[n] * h[n]$ & $y[4] = 5$, $y[14] = 0$ $N = ?$

$$y[n] = x[n] * h[n] = h[n] * x[n] = \sum_k h[k]\, x[n-k]$$

$$= \sum_{k=0}^{N} x[n-k]$$

$$y[n] = \sum_{k=0}^{N} x[n-k]$$ and we know $y[4] = 5$ & $y[14] = 0$

$$y[4] = \sum_{k=0}^{N} x[4-k] = \underbrace{x[4] + x[3] + x[2] + x[1] + x[0]}_{\text{all "1" } (\Sigma = 5)} + \underbrace{x[-1] + \cdots + x[4-N]}_{\text{this terms must be all zero}}$$

$\Rightarrow \; 4 - N \ge 9$ & $4 - N \le 0 \Rightarrow$ $N \ge 4$ & $N \le -5$ ↓ not possible

for $y[14] = 0$

$$0 = \sum_{k=0}^{N} x[14-k] = x[14] + x[13] + \cdots + x[10] + \underbrace{x[9] + \cdots + x[14-N]}_{\text{these terms are } > 0}$$

hence $14 - N \ge 10$ $\boxed{N \le 4}$

as a result $N \le 4$ & $N \ge 4 \Rightarrow$ hence $\boxed{N = 4}$

(4)

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k] \quad ; \; g[n] = u[n] - u[n-4]$$

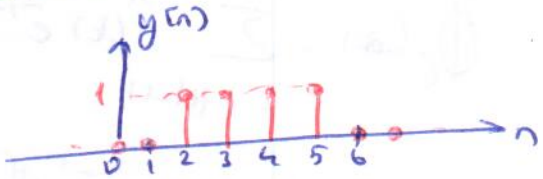
a) $x[n] = \delta[n-1]$

$$\Rightarrow y[n] = \sum_k \delta[k-1]\, g[n-2k] = \sum_k \delta[k-1]\,\big(u[n-2k] - u[n-2k-4]\big)$$

$$= \sum_k \delta[k-1]\, u[n-2k] - \sum_k \delta[k-1]\, u[n-2k-4]$$

when $k=1$ it has a value rather than '0' $\Rightarrow$ same

$$y[n] = u[n-2] - u[n-6]$$



b) $x[n] = \delta[n-2] \Rightarrow y[n] = \sum_k \delta[k-2]\,\big(u[n-2k] - u[n-2k-4]\big)$

$$= u[n-4] - u[n-8]$$

c) Linearity ?

$$x_1[n] \rightarrow y_1[n] = \sum_k x_1[k]\, g[n-2k]$$

$$x_2[n] \rightarrow y_2[n] = \sum_k x_2[k]\, g[n-2k]$$

$$x_3[n] = a x_1[n] + b x_2[n] \longrightarrow y_3[n] = \sum_k x_3[k]\, g[n-2k]$$

$$= \sum_k \big(a x_1[k] + b x_2[k]\big)\, g[n-2k]$$

$$= a \sum_k x_1[k]\, g[n-2k] + b \sum_k x_2[k]\, g[n-2k]$$

$$= a y_1[n] + b y_2[n] \Rightarrow \text{Hence; linear.}$$

TI ?

$$x[n] \rightarrow y[n] = \sum_k x[k]\, g[n-2k]$$

$$z[n] = x[n-n_0] \rightarrow y_1[n] = \sum_k z[k]\, g[n-2k] = \sum_k x[k-n_0]\, g[n-2k]$$

$$y[n-n_0] = \sum_k x[k]\, g[n-n_0-2k] \neq y_1[n] \quad \text{so}$$

Not Time invariant.

(4) cont'd.

d) $x[n] = u[n] \Rightarrow y[n] = \sum_k u[k]\,\underbrace{(u[n-2k] - u[n-2k-4])}$

$\delta[n-2k] + \delta[n-2k-1] + \delta[n-2k-2] + \delta[n-2k-3]$

$$\Rightarrow y[n] = \underbrace{\sum_k u[k]\,\delta[n-2k]}_{n-2k=0,\ k=n/2} + \underbrace{\sum_k u[k]\,\delta[n-2k-1]}_{n-2k-1=0,\ k=\frac{n-1}{2}} + \sum_k u[k]\,\delta[n-2k-2] + \sum_k u[k]\,\delta[n-2k-3]$$

$$y[n] = u[n/2] + u\left[\tfrac{n-1}{2}\right] + u\left[\tfrac{n-2}{2}\right] + u\left[\tfrac{n-3}{2}\right] = \begin{cases} 0 & ; n<0 \\ 1 & ; n=0,1 \\ 2 & ; n \geq 2 \end{cases}$$

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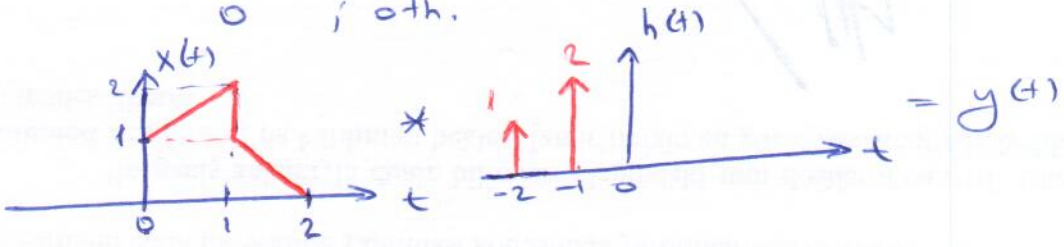
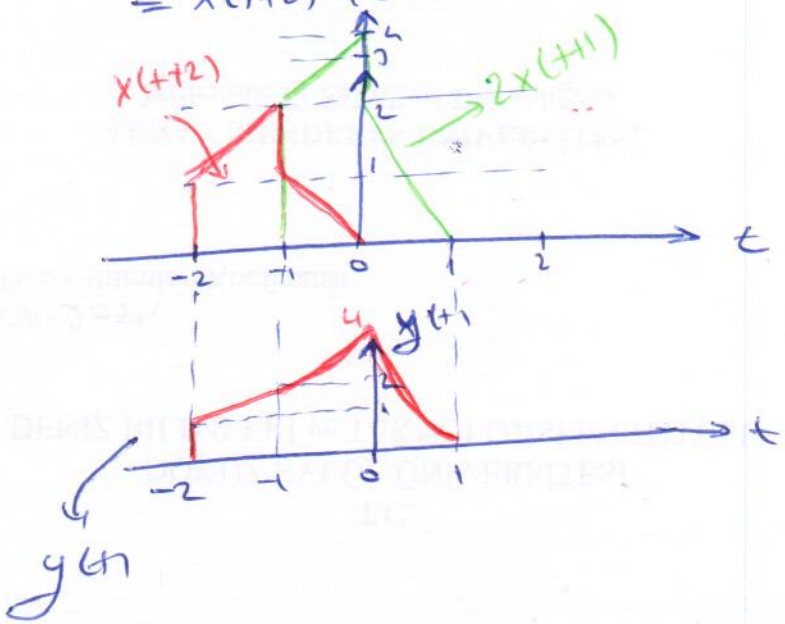
(5) $x(t) = \begin{cases} t+1 & ; 0 \leq t \leq 1 \\ 2-t & ; 1 < t \leq 2 \\ 0 & ; \text{o.th.} \end{cases}$ $\quad h(t) = \delta(t+2) + 2\delta(t+1)$

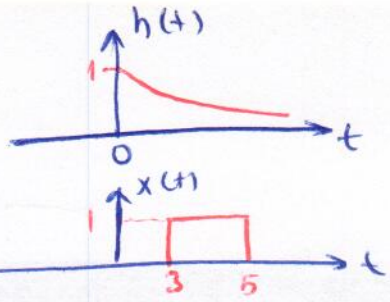


$y(t) = x(t) * h(t) = x(t) * (\delta(t+2) + 2\delta(t+1))$

$= x(t+2) + 2x(t+1)$



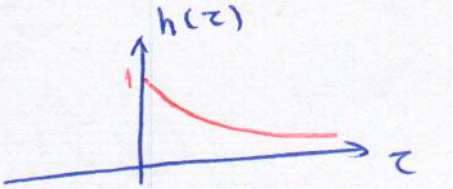
(6) $x(t) = u(t-3) - u(t-5)$ & $h(t) = e^{-3t}u(t)$

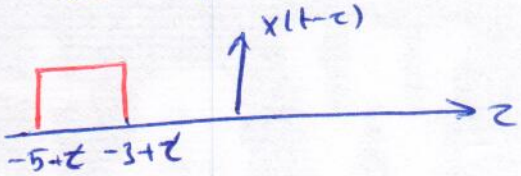


a) $y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$= \int_{3}^{5} e^{-3(t-\tau)}u(t-\tau)\,d\tau \quad ?$$





$t-3<0 \Rightarrow t<3 \Rightarrow y(t)=0$

$t \geq 3 \Rightarrow y(t) = \int_{t-5}^{t-3} e^{-3\tau}\,d\tau = \frac{e^{-3\tau}}{-3}\Big|_{t-3}^{t-5} = -\frac{1}{3}\left(e^{-3(t-5)} - e^{-3(t-3)}\right)$

$$= \frac{e^{-3t+9} - e^{-3t+15}}{3}$$

$$y(t) = \begin{cases} 0 & ; t<3 \\ e^{-3t}\left(\frac{e^{9}-e^{15}}{3}\right) & ; t \geq 3 \end{cases}$$

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b) $g(t) = \left(\frac{d}{dt}x(t)\right) * h(t)$ $\qquad \frac{d}{dt}x(t) = \delta(t-3) - \delta(t-5)$

$g(t) = \left(\delta(t-3) - \delta(t-5)\right) * h(t) = h(t-3) - h(t-5)$

$= e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5)$

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c) $g(t) \sim y(t)$?

(7) $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$ $\quad y(t) = x(t) * h(t)$

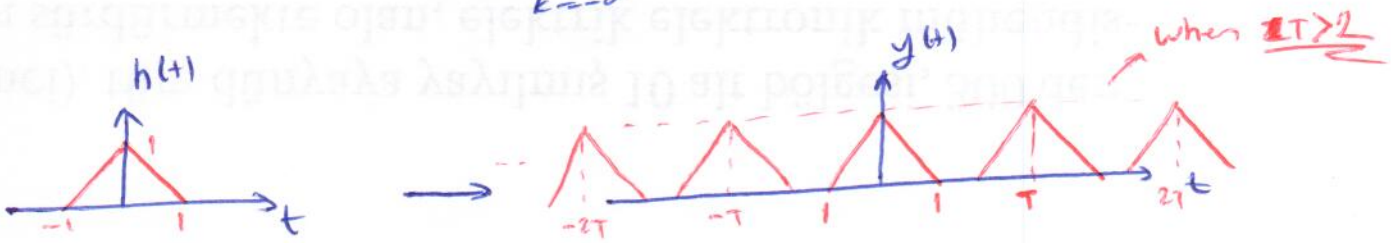
$$y(t) = x(t) * h(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

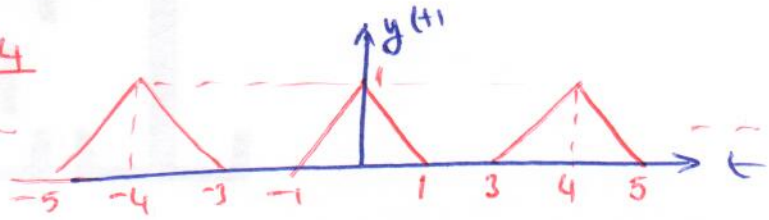
$$= \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(\tau-kT)\, h(t-\tau)\, d\tau$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \underbrace{\delta(\tau-kT)}_{\tau=kT} h(t-\tau)\, d\tau$$

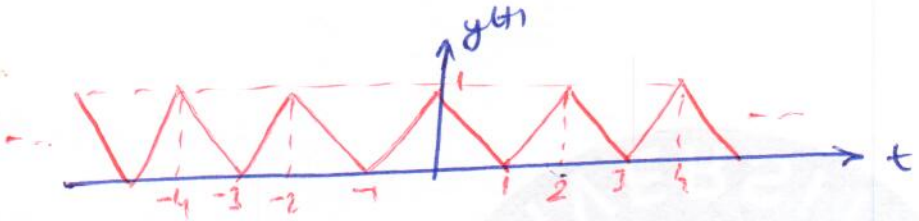
$$= \sum_{k=-\infty}^{\infty} h(t-kT)$$



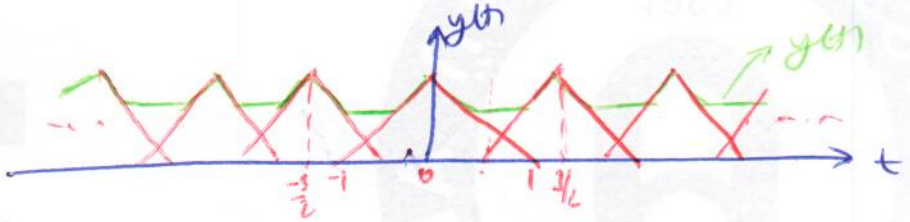
a) $T = 4$



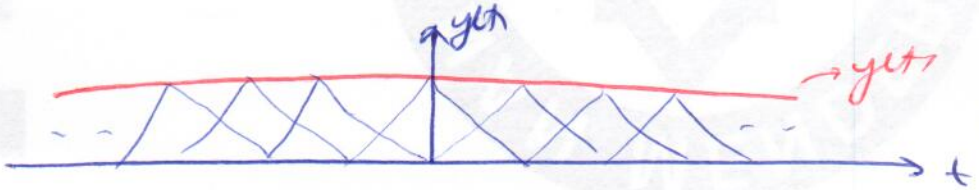
b) $T = 2$



c) $T = 1/2$



d) $\Rightarrow T = 1$



aliasing !

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Cassoc !!

$x[n] \rightarrow \boxed{h_1[n]} \rightarrow \boxed{h_2[n]} \rightarrow \boxed{h_2[n]} \rightarrow y[n] \equiv x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

$$h[n] = h_1[n] * \overbrace{h_2[n] * h_2[n]}^{g[n]} \leftarrow$$

$$h_2[n] = u[n] - u[n-2]$$



$g[n] = h_2[n] * h_2[n]$



$$h[n] = h_1[n] * g[n] = h_1[n] * \{\delta[n] + 2\delta[n-1] + \delta[n-2]\}$$
$$= h_1[n] + 2h_1[n-1] + h_1[n-2]$$

$n < 0 \Rightarrow h[n] = 0 \Rightarrow h_1[n] + 2h_1[n-1] + h_1[n-2] = 0$

$\boxed{h_1[n] = 0 \; ; n < 0 \text{ (for causality).}}$

$n = 0 \Rightarrow h[0] = 1 \Rightarrow h_1[0] + 2\cancel{h_1[-1]}^{0} + \cancel{h_1[-2]}^{0} = 1 \Rightarrow \underline{h_1[0] = 1}$

$n = 1 \Rightarrow h[1] = 5 \Rightarrow h_1[1] + 2\underbrace{h_1[0]}_{1} + \cancel{h_1[-1]}^{0} = 5 \Rightarrow \underline{h_1[1] = 3}$

$n = 2 \rightarrow h_1[2] + 2\underbrace{h_1[1]}_{3} + \underbrace{h_1[0]}_{1} = 10 \Rightarrow \underline{h_1[2] = 3}$

$n = 3 \Rightarrow h_1[3] + 2\underbrace{h_1[2]}_{3} + \underbrace{h_1[1]}_{3} = 11 \Rightarrow \underline{h_1[3] = 2}$

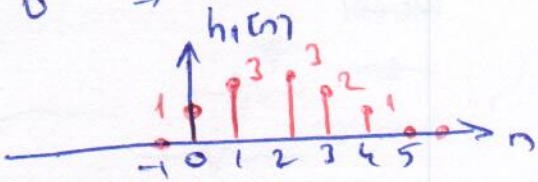
$n = 4 \Rightarrow h_1[4] + 2\underbrace{h_1[3]}_{2} + \underbrace{h_1[2]}_{3} = 8 \Rightarrow \underline{h_1[4] = 1}$

$n = 5 \Rightarrow h_1[5] + 2\underbrace{h_1[4]}_{1} + \underbrace{h_1[3]}_{2} = 4 \Rightarrow \underline{h_1[5] = 0}$

$n = 6 \Rightarrow h_1[6] + 2\underbrace{h_1[5]}_{0} + \underbrace{h_1[4]}_{1} = 1 \Rightarrow h_1[6] = 0$

$n = 7 \Rightarrow h_1[7] + 2\underbrace{h_1[6]}_{0} + \underbrace{h_1[5]}_{0} = 0 \Rightarrow h_1[7] = 0$

$\Rightarrow$



b) $x[n] = \delta[n] - \delta[n-1]$

$$y[n] = x[n] * h[n] = (\delta[n] - \delta[n-1]) * h[n] = h[n] - h[n-1]$$

$h[n] \rightarrow$	0	1	3	3	2	1	0	
$-\; h[n-1] \rightarrow$	0	0	1	3	3	2	1	0
$y[n] \rightarrow$	0	1	2	0	-1	-1	-1	0



(9)

h[n]	causal	stable
a	+	+
b	+	+
c	−	−
d	−	+
e	+	−
f	−	+
g	+	−

(10)

h[n]	causal	stable
a	+	+
b	−	−
c	−	+
d	−	+
e	−	+
f	+	−
g	+	−

(11) LTI , IRC $y[n]+2y[n-1] = x[n]+2x[n-2]$

$y[n] = x[n]+2x[n-2]-2y[n-1]$

for IRC assumption $y[n]=0 \; ; n<0$ $(y[-1]=y[-2]=\dots=0)$

$n=0; y[0] = x[0]+2x[-2]-2y[-1] = 3+2-0 = 5$

$n=1; y[1] = x[1]+2x[-1]-2y[0] = 2+2\times2-2\times5 = -4$

$n=2; y[2] = x[2]+2x[0]-2y[1] = 2+2\times3-2\times(-4) = 16$

$y[3] = x[3]+2x[1]-2y[2] = 1+2\times2-2(16) = -27$

$y[4] = x[4]+2x[2]-2y[3] = 0+2\times2-2\cdot(-27) = 58$

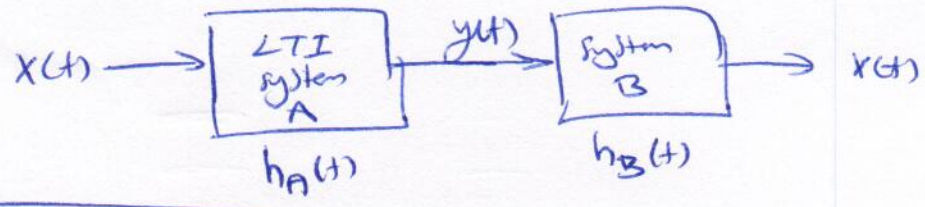
$y[5] = x[5]+2x[3]-2y[4] = 0+2\times1-2\cdot(58) = -114$

$y[6] = x[6]+2x[4]-2y[5] = 0+0-2\cdot(-114) = 228$

$y[7] = -2y[6] = -2\cdot(228)$

$y[n] = -2y[n-1] \; ; n \geq 5$

(12)



$h_A(t) * h_B(t) = \delta(t)$ inverse systems!!

$x_1(t) \rightarrow y_1(t)$
$x_2(t) \rightarrow y_2(t)$

a) we know that for system B $y(t) \rightarrow x(t)$

$y(t) = ay_1(t) + by_2(t) \xleftrightarrow{B} ax_1(t) + bx_2(t)$ (due to invertibility)

b) $y_1(t-\tau) \xleftrightarrow{B}$?

we know that system A is LTI it means

$x_1(t) \longleftrightarrow y_1(t)$
$x_1(t-\tau) \longleftrightarrow y_1(t-\tau)$ } TI property

and we know that system B is an inverse system for system A

$y_1(t) \xleftrightarrow{B} x_1(t)$
$y_1(t-\tau) \longleftrightarrow x_1(t-\tau)$ ←