EE 303

SIGNALS & SYSTEMS

Question Bank #1

Due: Not collected (for your self-study)

1) Determine the fundamental period of the signal $x(t) = 2\cos(10t + 1) - \sin(4t - 1)$.

2) Determine the fundamental period of the signal $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$.

3) Consider the discrete-time signal

$$x[n] = 1 - \sum_{k=3}^{\infty} \delta[n - 1 - k].$$

Determine the values of the integers M and n_0 so that $x[n]$ may be expressed as

$$x[n] = u[Mn - n_0].$$

4) Consider a system S with input $x[n]$ and output $y[n]$. This system is obtained through a series interconnection of a system S_1 followed by a system S_2. The input-output relationships for S_1 and S_2 are

$$S_1: \quad y_1[n] = 2x_1[n] + 4x_1[n - 1],$$

$$S_2: \quad y_2[n] = x_2[n - 2] + \frac{1}{2}x_2[n - 3],$$

where $x_1[n]$ and $x_2[n]$ denote input signals.

(a) Determine the input-output relationship for system S.

(b) Does the input-output relationship of system S change if the order in which S_1 and S_2 are connected in series is reversed (i.e., if S_2 follows S_1)?

5) Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is

$$y[n] = x[n]x[n - 2].$$

(a) Is the system memoryless?

(b) Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.

(c) Is the system invertible?

6) Consider a continuous-time system with input $x(t)$ and output $y(t)$ related by

$$y(t) = x(\sin(t)).$$

(a) Is this system causal?

(b) Is this system linear?

7) Consider a discrete-time system with input $x[n]$ and output $y[n]$ related by

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k],$$

where n_0 is a finite positive integer.

(a) Is this system linear?

(a) Is this system time-invariant?

(c) If $x[n]$ is known to be bounded by a finite integer B (i.e., $|x[n]| < B$ for all n), it can be shown that $y[n]$ is bounded by a finite number C. We conclude that the given system is stable. Express C in terms of B and n_0.

8) A continuous-time signal $x(t)$ is shown in Figure P1.21. Sketch and label carefully each of the following signals:

(a) $x(t-1)$ (b) $x(2-t)$ (c) $x(2t+1)$

(d) $x(4-\frac{t}{2})$ (e) $[x(t)+x(-t)]u(t)$ (f) $x(t)[\delta(t+\frac{3}{2})-\delta(t-\frac{3}{2})]$

9) A discrete-time signal is shown in Figure P1.22. Sketch and label carefully each of the following signals:

(a) $x[n-4]$ (b) $x[3-n]$ (c) $x[3n]$

(d) $x[3n+1]$ (e) $x[n]u[3-n]$ (f) $x[n-2]\delta[n-2]$

(g) $\frac{1}{2}x[n]+\frac{1}{2}(-1)^n x[n]$ (h) $x[(n-1)^2]$

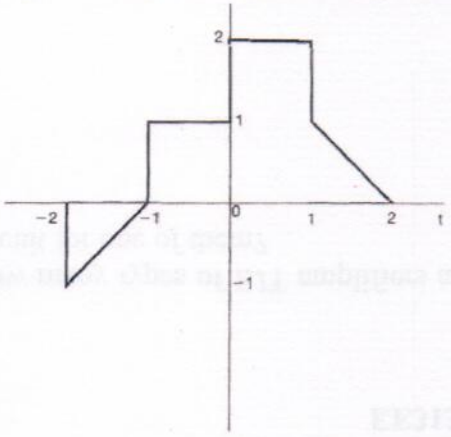


Figure P1.21

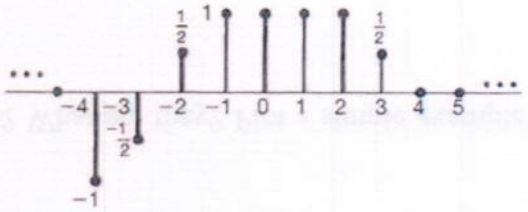


Figure P1.22

10) Consider a system S with input $x[n]$ and output $y[n]$ related by

$$y[n] = x[n]\{g[n] + g[n-1]\}.$$

(a) If $g[n] = 1$ for all n, show that S is time invariant.

(b) If $g[n] = n$, show that S is not time invariant.

(c) If $g[n] = 1 + (-1)^n$, show that S is time invariant.

11) Consider the feedback system of Figure P1.46. Assume that $y[n] = 0$ for $n < 0$.

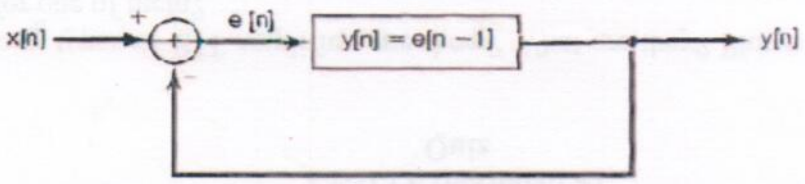


Figure P1.46

(a) Sketch the output when $x[n] = \delta[n]$.

(b) Sketch the output when $x[n] = u[n]$.

EE303 – Question Bank #1 — 18/19 Fall

(1) $x(t) = \underbrace{2\cos(10t+1)}_{} - \underbrace{\sin(4t-1)}_{}$

for periodicity $\Rightarrow 10(t+T_1)$ $\quad$ $4(t+T_2) = 4t + 2\pi k$

$\hookrightarrow 10T_1 = 2\pi k$ $\quad$ $\hookrightarrow 4T_2 = 2\pi$

$T_1 = \frac{2\pi}{10}$ $\quad$ $T_2 = \frac{2\pi}{4}$

first term is periodic by $2\pi/10 = \pi/5$, the second term is also periodic by $2\pi/4 = \pi/2$ $\Rightarrow$ Then, $x(t)$ is periodic by $LCM(\pi/5, \pi/2) = \pi = T_0$

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(2) $x[n] = 1 + e^{j4\pi n/7} + e^{j\pi n/5} \Rightarrow N_0 = ?$

$e^{j4\pi n/7} = e^{j4\pi(N_1+n)/7}$ $\quad$ $e^{\frac{j\pi n}{5}} = e^{\frac{j\pi(n+N_2)}{5}}$

$\Rightarrow e^{\frac{j4\pi N_1}{7}} \triangleq 1$ (for periodicity) $\quad$ $e^{j\pi N_2/5} \triangleq 1$ (for periodicity)

$\Rightarrow \frac{4\pi N_1}{7} = 2\pi k$ $\quad$ $\Rightarrow \frac{\pi N_2}{5} = 2\pi m$

$\frac{N_1}{k} = \frac{7}{2} \Rightarrow$ for $k=2$ $\quad$ $\frac{N_2}{m} = 10 \Rightarrow$ for $m=1$

$\boxed{N_1 = 7}$ $\quad$ $\boxed{N_2 = 10}$

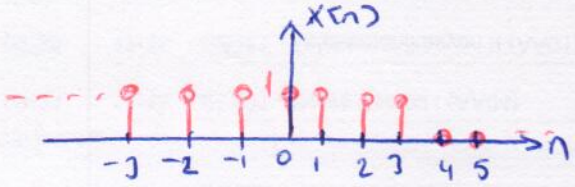
hence $x[n]$ has a fundmental period of $N_0 = LCM(N_1, N_2)$

$= LCM(7, 10)$

$= 70$

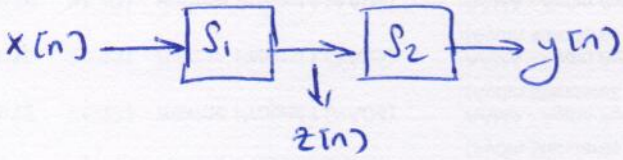
3) $x[n] = 1 - \sum_{k=3}^{\infty} \delta[n-1-k] \overset{?}{\Longrightarrow} x[n] = u[Mn - n_0]$

$x[n] = 1 - \delta[n-4] - \delta[n-5] - \delta[n-6] - \cdots$



$\Rightarrow$ This signal is $u[-n+3]$
hence $M = -1$, $n_0 = -3$

4)



$S_1: y_1[n] = 2x_1[n] + 4x_1[n-1]$

$S_2: y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3]$

a) for $S_1 \Rightarrow z[n] = 2x[n] + 4x[n-1]$

for $S_2 \Rightarrow y[n] = z[n-2] + \frac{1}{2}z[n-3]$

$= \underbrace{2x[n-2] + 4x[n-3]}_{z[n-2]} + \underbrace{x[n-3] + 2x[n-4]}_{\frac{1}{2}z[n-3]}$

* $y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$ i/o relationship for System "S"

b) If the change the order; $x[n] \rightarrow [S_2] \rightarrow [S_1] \rightarrow y[n]$, $k[n]$ between S_2 and S_1

$k[n] = x[n-2] + \frac{1}{2}x[n-3]$

$y[n] = 2k[n] + 4k[n-1]$

$\longrightarrow y[n] = 2x[n-2] + 4x[n-3] + x[n-3] + 2x[n-4]$

* $y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$

NOT EFFECTED !!

5) $y[n] = x[n]x[n-2]$

a) O/p depend on the past samples of input, hence System HAS Memory!

b) $x[n] = A\delta[n] \rightarrow y[n] = A\delta[n] \cdot A(\delta[n-2])$

$= A^2 \delta[n] \cdot \delta[n-2]$

$= 0$

c) In part a we found that for some inputs, output is zero. Hence, we cannot obtain that input by observing zero output. Hence, system is NOT INVERTIBLE !!

6) $y(t) = x(\sin(t))$

a) Causality $\Rightarrow$ for $t = -\pi \Rightarrow y(-\pi) = x(\sin(-\pi)) = x(0)$

so, output depends future values of input $\Rightarrow$ No Causal.

b) Linearity $\rightarrow$ $x_1(t) \rightarrow y_1(t) = x_1(\sin(t))$

$x_2(t) \rightarrow y_2(t) = x_2(\sin t)$

$x_3(t) = a x_1(t) + b x_2(t) \Rightarrow y_3(t) = x_3(\sin(t)) = a x_1(\sin(t)) + b x_2(\sin(t))$

$= a y_1(t) + a_2 y_2(t) \Rightarrow$ hence LINEAR

7) $y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$ n_0 is positive integer!

a) $y[n] = x[n-n_0] + x[n-n_0+1] + \cdots + x[-1] + x[0] + x[1] + \cdots + x[n+n_0]$

↳ As this is a simple adder (adding/summation operation) it is Linear.

Check 1.

$x_1[n] \rightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$

$x_2[n] \rightarrow y_2[n] = \sum_{k=\cdots} x_2[k]$

$x_3[n] = a x_1[n] + b x_2[n] \longrightarrow y_3[n] = \sum_{k=n-n_0}^{n+n_0} x_3[k] = \sum_{k=n-n_0}^{n+n_0} a x_1[k] + b x_2[k]$

$= a \sum_{k=\cdots}^{n+n_0} x_1[k] + b \sum_{k=\cdots}^{n+n_0} x_2[k] = a y_1[n] + b y_2[n]$ L

b) $x_m[n] = x[n-n_0]$

$$y_m[n] = \sum_{k=n-n_0}^{n+n_0} x_m[k] = \sum_{k=n-n_0}^{n+n_0} x[k-n_0]$$

$$y[n-n_0] = ? \Rightarrow y[n-n_0] = \sum_{k=(n-n_0)-n_0}^{(n-n_0)+n_0} x[k] = \sum_{k=n-2n_0}^{n} x[k]$$

$y[n-n_0] \neq y_m[n] \Rightarrow$ Hence, this system is time-varying

c) $|x[n]| < B$, $|y[n]| < C$

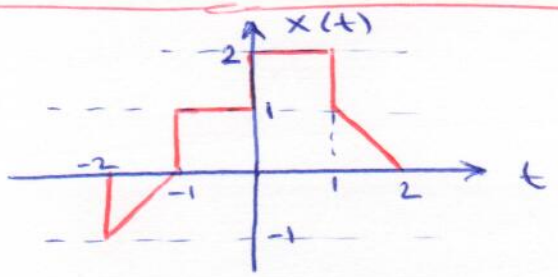
$$y[n] = \sum_{k=n-n_0}^{n+n_0} x(k) \Rightarrow |y[n]| = \left| \sum_{k=n-n_0}^{n+n_0} x(k) \right| \leq \sum_{k=n-n_0}^{n+n_0} |x[k]|$$

$$< \sum_{k=n-n_0}^{n+n_0} B$$

$$< B(2n_0+1)$$

$|y[n]| < C$
$|y[n]| < B(2n_0+1)$
} hence $C = B(2n_0+1)$

8)

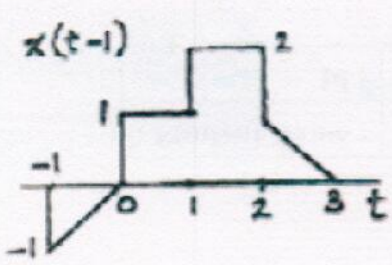


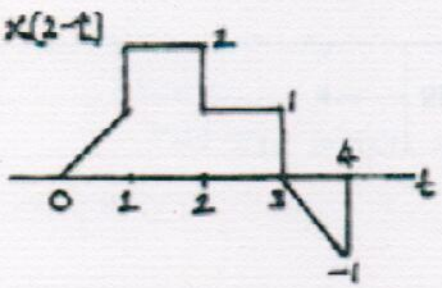
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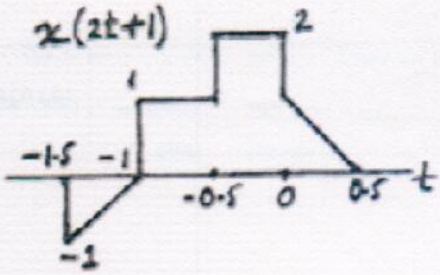
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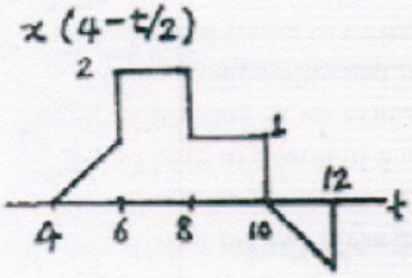
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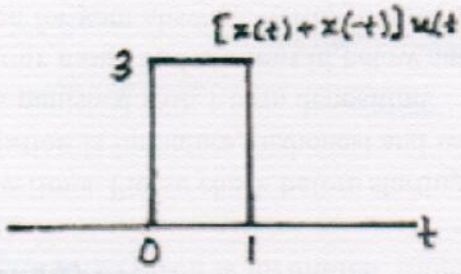
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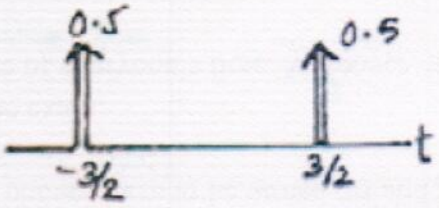




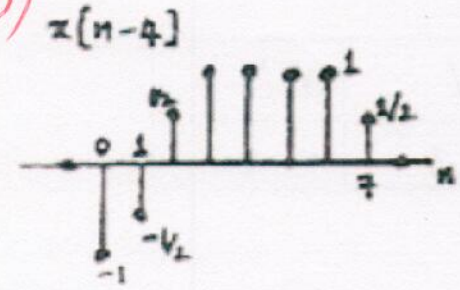




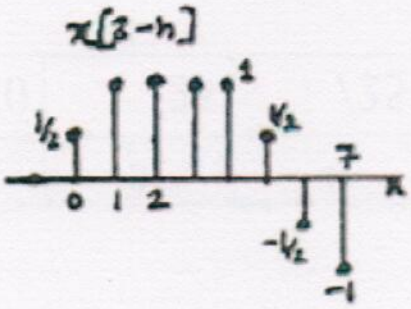




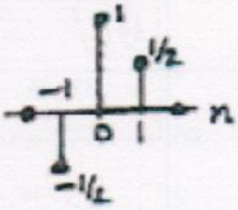
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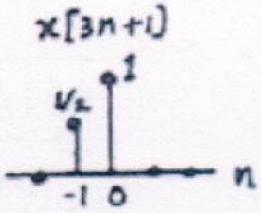
(a)



(b)

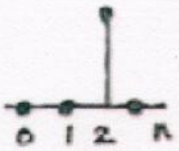


(c)



(d)

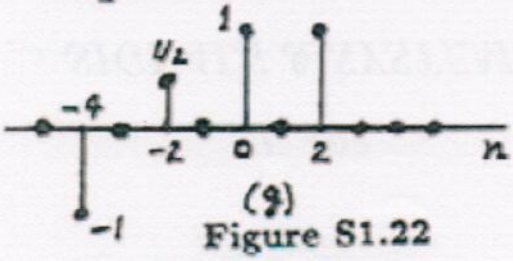
x[n]·u[n-3]
= x[n]



(e)

(f)

$\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$



(g)

x[(n-1)²]



(h)

Figure S1.22

10) $x[n] \rightarrow [S] \rightarrow y[n]$

$$y[n] = x[n]\{g[n] + g[n-1]\}$$

a) $g[n] = 1 \ \forall n \Rightarrow y[n] = x[n](1+1) = 2x[n]$

$x[n] \rightarrow y[n] = 2x[n]$

$x[n-n_0] \rightarrow z[n] = 2x[n-n_0]$

$y[n]\big|_{n=n-n_0} = y[n-n_0] = 2x[n-n_0] = z[n]$ $\Rightarrow$ hence S is time-invariant

b) $g[n] = n \Rightarrow y[n] = x[n]\{n + n-1\} = (2n-1)x[n]$

$x[n] \rightarrow y[n] = (2n-1)x[n]$

$x[n-n_0] \rightarrow z[n] = (2n-1)x[n-n_0] = (2n-1)x[n-n_0]$

$y[n]\big|_{n=n-n_0} = y[n-n_0] = (2(n-n_0)-1)x[n-n_0]$

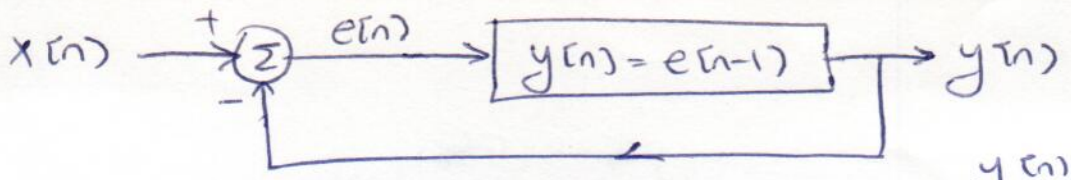
$= (2n-2n_0-1)x[n-n_0) \neq z[n]$ $\Rightarrow$ Hence, Now Time-varying

c) $g[n] = 1 + (-1)^n \Rightarrow g[n] = \begin{cases} 2 & ; n \text{ is even} \\ 0 & ; n \text{ is odd} \end{cases}$

$y[n] = \{x[n]\{\underbrace{g[n]}_{\text{this only 2}} + g[n-1]\}$ $\Rightarrow y[n] = 2x[n]$ as in part "a"

this system is TI

11)



$y[n] = 0 \quad ; n < 0$

a) ~~$\delta[n] \Rightarrow x[n] = \delta[n]$~~

$y[n] = e[n-1]$ $\qquad$ $e[n] = x[n] - y[n]$

$$\boxed{y[n] = x[n-1] - y[n-1]}$$

a) $x[n] = \delta[n] \Rightarrow y[n] = \delta[n-1] - y[n-1]$

$n < 0 \quad y[n] = 0$

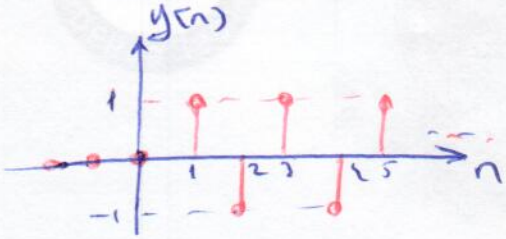
$n = 0 \quad y[0] = \delta[-1] - y[-1] = 0$

$n = 1 \quad y[1] = \delta[0] - y[0] = 1$

$n = 2 \quad y[2] = \delta[1] - y[1] = -1$

when $n \geq 2 \quad \delta[n] = 0 \Rightarrow y[n] = -y[n-1]$

$$y[n] = \begin{cases} 0 & ; n \leq 0 \\ (-1)^{n+1} & ; n > 0 \end{cases}$$



b) $x[n] = u[n] \rightarrow y[n] = u[n-1] - y[n-1]$

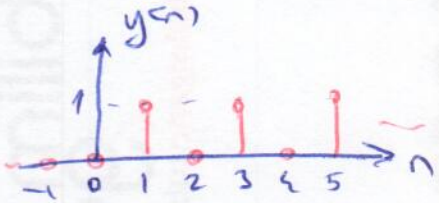
$n = 0 \; ; \; y[0] = u[-1] - y[-1] = 0$

$n = 1 \; ; \; y[1] = u[0] - y[0] = 1$

$n = 2 \; ; \; y[2] = u[1] - y[1] = 1 - 1 = 0$

$n = 3 \; ; \; y[3] = u[2] - y[2] = 1 - 0 = 1$

$n = 4 \; ; \; y[4] = u[3] - y[3] = 1 - 1 = 0$



$$y[n] = \begin{cases} 0 & ; n \leq 0 \\ 1 & ; n \text{ is odd} \\ 0 & ; n \text{ is even} \end{cases}$$