



AYDIN ADNAN MENDERES UNIVERSITY FACULTY OF ENGINEERING

Department of Electrical and Electronics Engineering

EE303 – Signals and Systems

Prof. Dr. Yılmaz KALKAN



1. Signals & Systems (Chapter 1 of Oppenheim – 6 Hrs.)

Introduction, Continuous-Time and Discrete-Time Signals, Periodicity, Even and Odd Signals, Exponential and Sinusoidal Signals, the Unit Impulse and Unit Step Functions, Continuous-Time and Discrete-Time Systems, Basic System Properties (Memory, Invertibility, Causality, Stability, Time Invariance, Linearity)

2. Linear Time-Invariant (LTI) Systems (Chapter 2 of Oppenheim – 9 Hrs.)

Discrete-Time LTI Systems, Convolution, Continuous-Time LTI Systems, Properties of LTI Systems, LTI Systems with and without Memory, Invertibility of LTI Systems, Causality for LTI Systems, Stability for LTI Systems, Unit Step Response of an LTI System, Causal LTI Systems Described by Differential and Difference Equations, Linear Constant-Coefficient Differential Equations

3. Fourier Series (Chapter 3 of Oppenheim – 6 Hrs.)

Fourier Series Representation of CT Periodic Signals, Convergence of Fourier Series, Gibs Phenomenon, Properties of CT Fourier Series, Fourier Series Representation of DT Periodic Signals, Properties of DT Fourier Series, Fourier Series and LTI Systems

4. CT Fourier Transform (Chapter 4 of Oppenheim – 6 Hrs.)

Op-Amp Terminals & Representation of Aperiodic CT Signals, Convergence of CT Fourier Transforms, Fourier Transform for Periodic Signals, Properties of CT Fourier Transform, Convolution Property, Systems Characterized by Linear Constant-Coefficient Differential Equations

5. DT Fourier Transform (Chapter 5 of Oppenheim – 6 Hrs.)

Representation of Aperiodic DT Signals, Convergence of DT Fourier Transforms, Fourier Transform for Periodic Signals, Properties of DT Fourier Transform, Convolution Property, Systems Characterized by Linear Constant-Coefficient Differential Equations

6. The Laplace Transform (Chapter 9 of Oppenheim – 6 Hrs.)

Introduction, The Region of Convergence for Laplace Transforms, The Inverse Laplace Transform, Pole-Zero Plot, First-Order and Second-Order Systems, Properties of the Laplace Transform, Analysis and Characterization of LTI Systems Using the Laplace Transform, The Unilateral Laplace Transform

7. The Z-Transform (Chapter 10 of Oppenheim – 6 Hrs.)

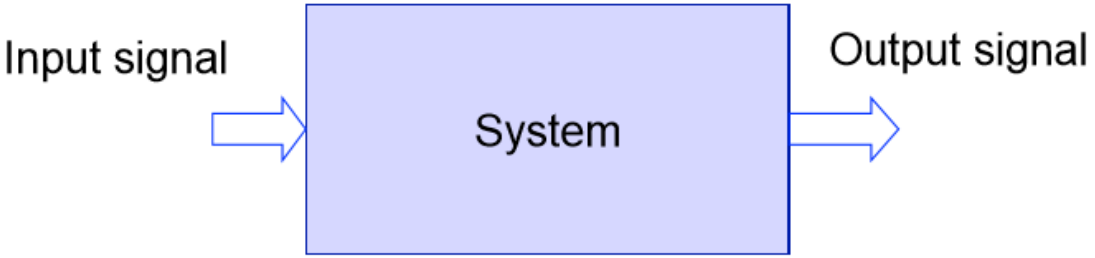
Introduction, The Region of Convergence for the Z-Transforms, The Inverse Z-Transform, Pole-Zero Plot, First-Order and Second-Order Systems, Properties of the Z-Transform, Analysis and Characterization of LTI Systems Using the Z-Transform, The Unilateral Z-Transform

Ch1: Signals & Systems – Introduction

What is a signal?

- A signal is a set of **information** of data.
- Any kind of physical variable subject to variations represents a signal.
- Signals are represented mathematically as functions of one or more independent variables.
  - A speech signal can be represented mathematically by acoustic pressure as a function of time,
  - A picture can be represented by brightness as a function of two spatial variables.

What is a system?



https://www.youtube.com/watch?v=EErkgr1MWw0

Ch1: Signals & Systems – Classification of signals

Classification of signals

- Continuous time – Discrete time ,
- Analog – Digital (numerical) ,
- Periodic – Aperiodic ,
- Energy – Power ,
- Deterministic – Random (probabilistic)
- ……

Note that;

– Such classes are not disjoint, so there are digital signals that are periodic of power type and others that are aperiodic of power type etc.

– Any combination of single features from the different classes is possible

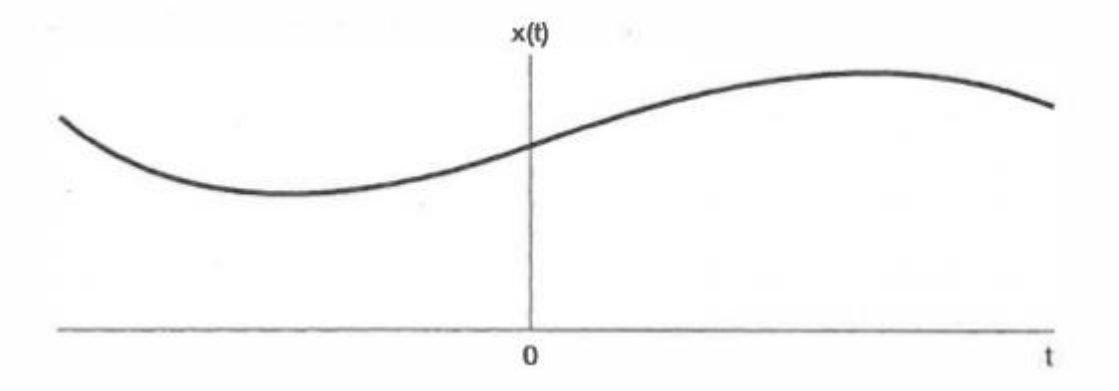
Ch1: Signals & Systems – Classification of signals

Continuous time (CT) – Discrete time (DT)

- **Continuous time signal:** a signal that is specified for every real value of the independent variable

– The independent variable is continuous, that is it takes **any value** on the real axis

– The domain of the function representing the signal has the cardinality of **real numbers**



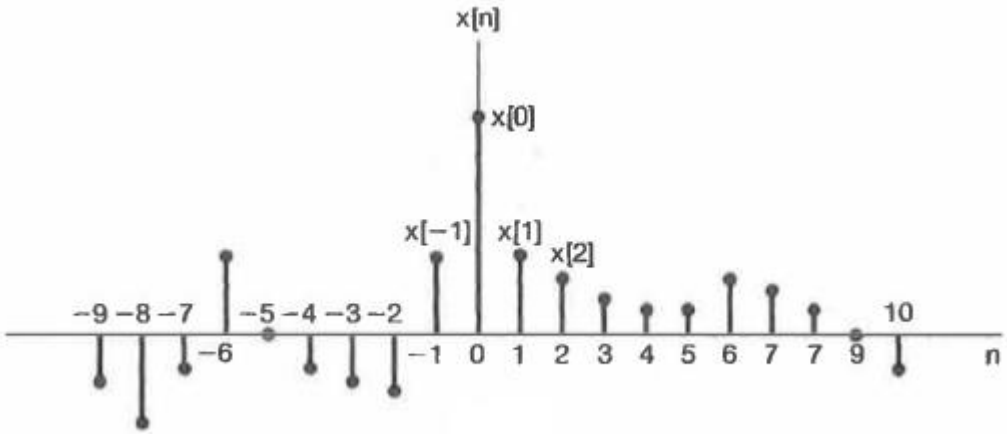
Ch1: Signals & Systems – Classification of signals

Continuous time (CT) – Discrete time (DT)

- **Discrete time signal:** a signal that is specified only for discrete values of the independent variable

– It is usually generated by sampling, so it will only have values at **equally spaced intervals** along the time axis

– The domain of the function representing the signal has the cardinality of **integer numbers**

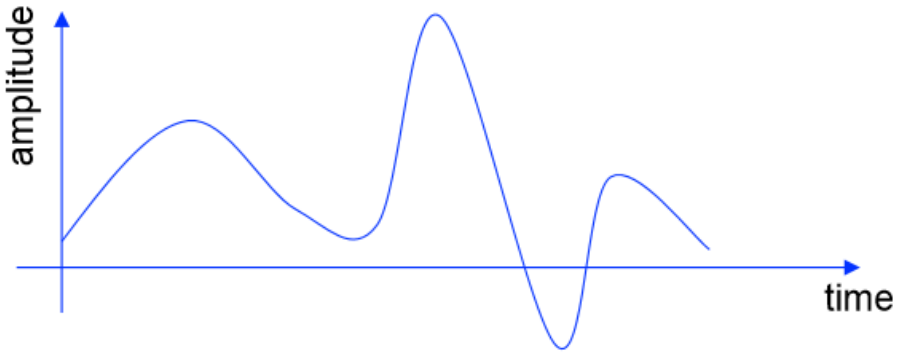


Ch1: Signals & Systems – Classification of signals

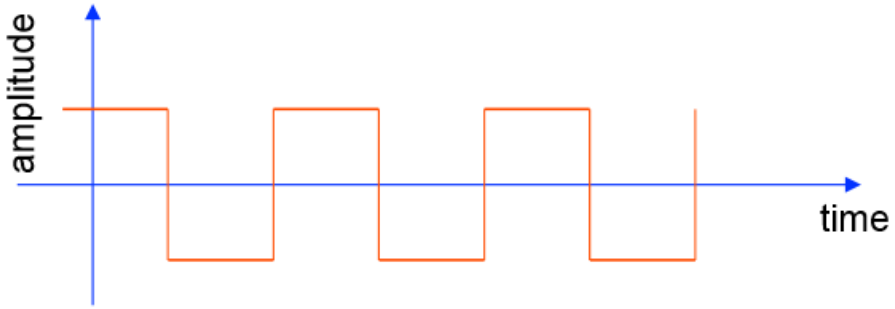
Analog Signal – Digital Signal

Example

- Continuous time analog



- Continuous time digital (or quantized)
  - binary sequence, where the values of the function can only be one or zero.

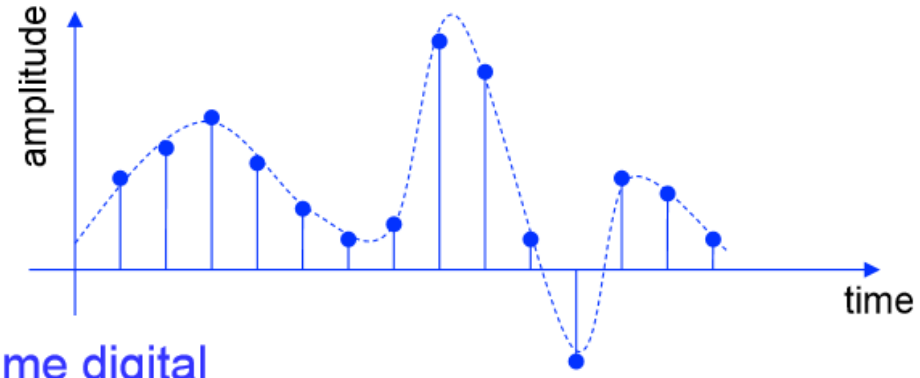


Ch1: Signals & Systems – Classification of signals

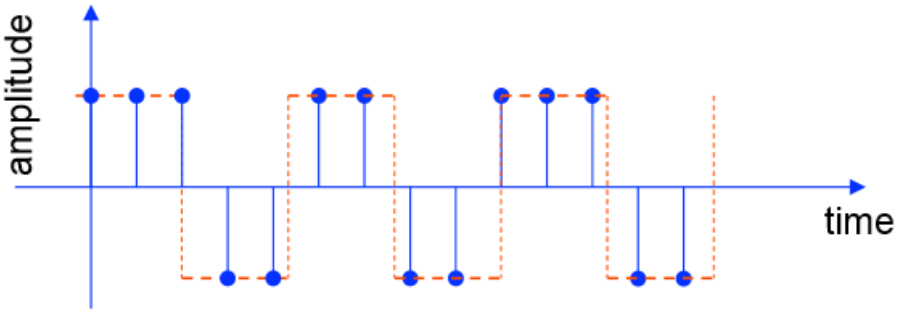
Analog Signal – Digital Signal

Example

- Discrete time analog



- Discrete time digital
  - binary sequence, where the values of the function can only be one or zero.



Ch1: Signals & Systems – Classification of signals

Periodic and Aperiodic Signals

- A signal f(t) is *periodic* if there exists a positive constant T_0 such that

$$f(t+T_0) = f(t) \qquad \forall t$$

- The *smallest* value of T_0 which satisfies such relation is said the *period* of the function f(t)
- A periodic signal remains unchanged when *time-shifted* of integer multiples of the period
- Therefore, by definition, it starts at minus infinity and lasts forever

$$-\infty \le t \le +\infty$$

$$-\infty \le n \le +\infty$$

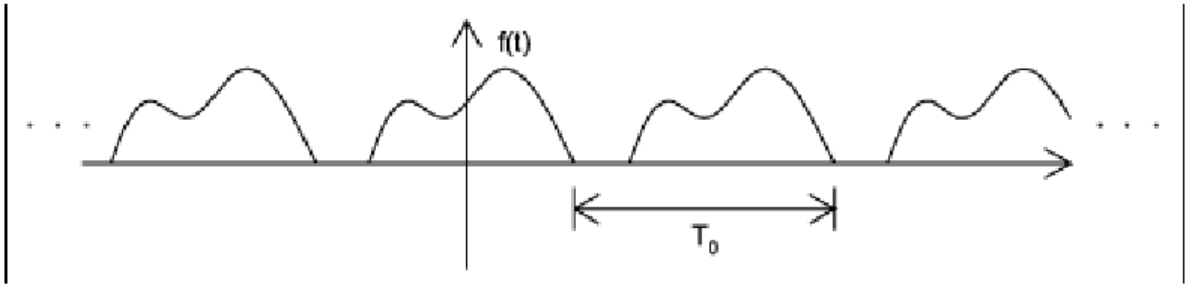
- Periodic signals can be generated by *periodical extension*

Ch1: Signals & Systems – Classification of signals

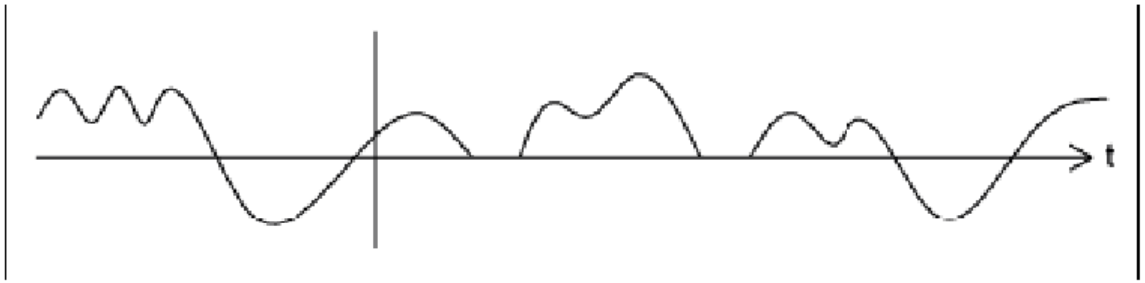
Periodic and Aperiodic Signals

Examples

- Periodic signal with period T_0



- Aperiodic signal



$$e^{j\frac{\pi}{2}} = \cos\left(\frac{pi}{2}\right) + jsin\left(\frac{pi}{2}\right) = 0 + 1j$$

Ex: Determine whether the signals below are periodic or aperiodic. Find their fundamental period if they are periodic.

a) $x_1(t) = je^{j10t} = e^{j\left(10t+\frac{\pi}{2}\right)} \Longrightarrow x_1(t) = e^{j(\omega t+\theta)} \Longrightarrow$ It is periodic

$$x_1(t) = x_1(t + T_1)$$

$$e^{j\left(10t+\frac{\pi}{2}\right)} = e^{j\left(10(t+T_1)+\frac{\pi}{2}\right)} = e^{j\left(10t+\frac{\pi}{2}\right)} . e^{j10T_1}$$

$$\Rightarrow \quad e^{j10T_1} = 1 \quad \Rightarrow \quad 10T_1 = 2\pi \quad \Rightarrow \quad T_1 = \frac{\pi}{5} \text{ s (fundamental period)}$$

For periodicty ⇒

$T_1 = \frac{2\pi}{W_1}$ should be a real number & T_1 is fundamental period

Euler's Formula

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

Ex: Determine whether the below signals are periodic or aperiodic. Find their fundamental period if they are periodic.

b) $x_2(t) = e^{(-1+j)t} = e^{-t}.e^{jt}$

For periodicity; $x_2(t) = x_2(t+T_2)$ must be ensured.

$$e^{-t}.e^{jt} = e^{-(t+T_2)}.e^{j(t+T_2)} = e^{-t}.e^{jt}.e^{-T_2}.e^{jT_2}$$

$$\Rightarrow \quad e^{-T_2}.e^{jT_2} = 1$$

$$\Rightarrow \quad e^{(-1+j)T_2} = e^{j2\pi}$$

$$\Rightarrow \quad T_2 = \frac{j2\pi}{j-1}$$

But $T_2 \in \mathbb{R}$ must be ensured. So; $x_2(t)$ signal **is not periodic**.

Ex: Determine whether the below signals are periodic or aperiodic. Find their fundamental period if they are periodic.

c) $x_3(t) = 2\cos(10t+1) - \sin(4t-1)$

$\omega_{31} = 10 \qquad \omega_{32} = 4$

$T_{31} = \frac{2\pi}{\omega_{31}} = \frac{\pi}{5} \qquad T_{32} = \frac{2\pi}{\omega_{32}} = \frac{\pi}{2}$

$$T_3 = \text{lcm}(T_{31}, T_{32}) = \text{lcm}\left(\frac{\pi}{5}, \frac{\pi}{2}\right) = \text{lcm}\left(\frac{2\pi}{10}, \frac{5\pi}{10}\right) = \frac{\pi}{10}\text{lcm}(2,5) = \pi \text{ s}$$

Ch1: Signals & Systems – Classification of signals

Periodic and Aperiodic Signals – Discrete Time

- A signal f(t) is *periodic* if there exists a positive constant T_0 such that

$$f(t+T_0) = f(t) \qquad \forall t$$

- Similarly, in DT, a signal x[n] is periodic if there exists a positive constant N suct that;

$$x[n+N] = x[n]; \quad \forall n$$

Ex: Determine whether the signals below are periodic or aperiodic. Find their fundamental period if they are periodic.

a) $x_1[n] = e^{j(2\pi/3)n}$

$x_1[n] = ? \; x_1[n+N] \Longrightarrow$ It is periodic

$$x_1[n] = e^{j(2\pi/3)n} \Longrightarrow e^{j(2\pi/3)(n+N)} = e^{j(2\pi/3)n} \cdot e^{j(2\pi/3)N} = x_1[n+N]$$

$$\Rightarrow e^{j(2\pi/3)N} = 1 \quad \Rightarrow (2\pi/3)N = 2\pi k \quad \Rightarrow \quad \frac{N}{k} = 3$$

$$\Rightarrow e^{j2\pi k} = 1$$

$$\Rightarrow \quad N = 3$$

If you can find such 'N' & 'k' where both are integer, then it is periodic by N. (N=3,6,9.,,,)

For periodicty ⇒

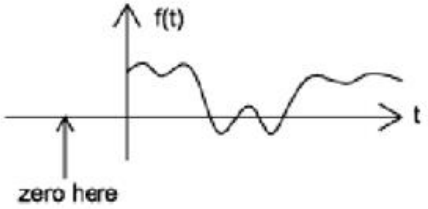
$\frac{N}{k} = \frac{2\pi}{W}$ and N and k must be integer. min(N) is fundamental period

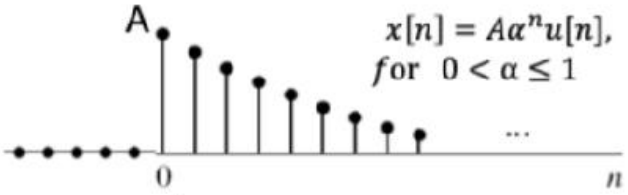
Ch1: Signals & Systems – Classification of signals

Causal and non-Causal Signals

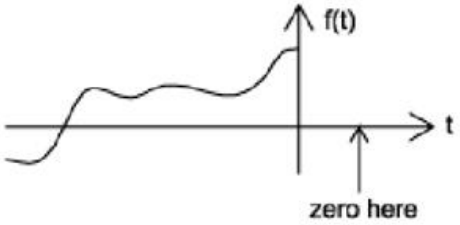
CT & **DT**

- *Causal* signals are signals that are *zero for all negative time (or spatial positions)*, while

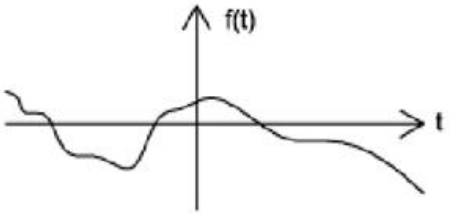


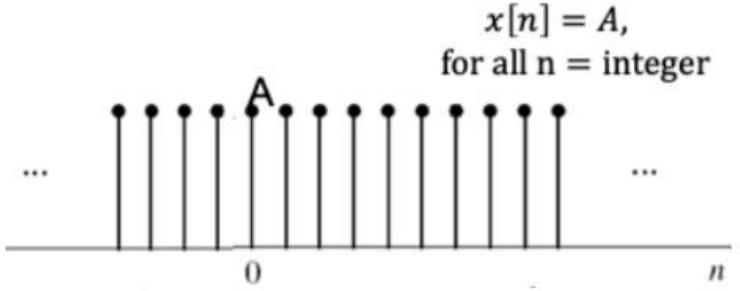


- *Anticausal* are signals that are zero for all *positive time (or spatial positions)*.



- *Noncausal* signals are signals that have nonzero values in both positive and negative time

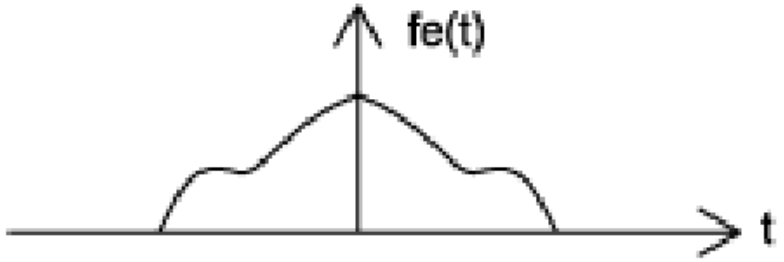




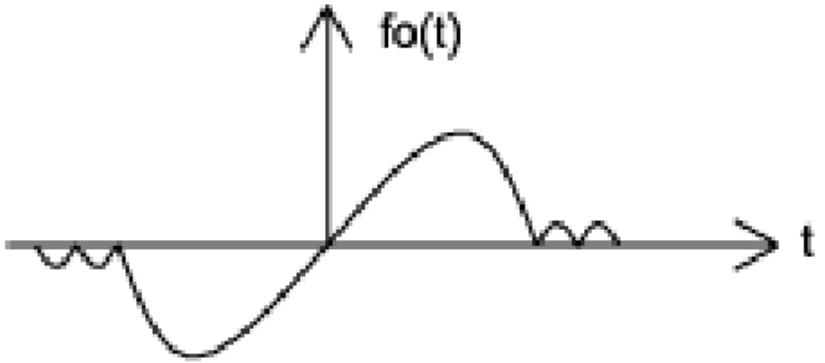
Ch1: Signals & Systems – Classification of signals

Even and Odd Signals

- An even signal is any signal f such that f (t) = f (-t). Even signals can be easily spotted as they are symmetric around the vertical axis.



- An odd signal, on the other hand, is a signal f such that f (t)= - (f (-t))



Ch1: Signals & Systems – Classification of signals

Decomposition in even and odd components

- Any signal can be written as a combination of an even and an odd signals
  - Even and odd components

$$f(t) = \frac{1}{2}\left(f(t) + f(-t)\right) + \frac{1}{2}\left(f(t) - f(-t)\right)$$

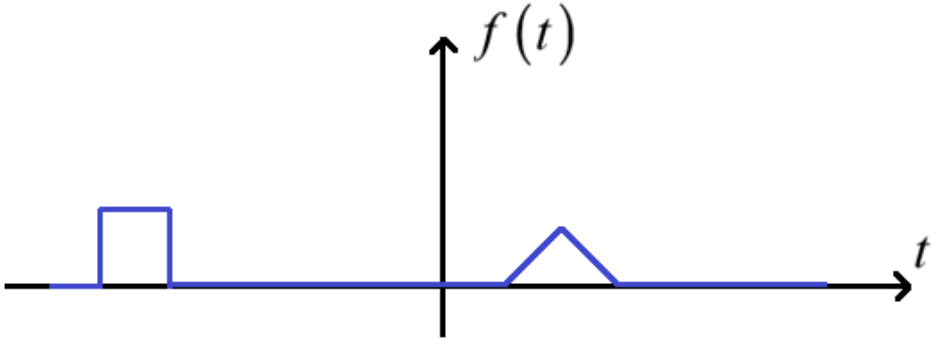
$$f_e(t) = \frac{1}{2}\left(f(t) + f(-t)\right) \quad \text{even component}$$

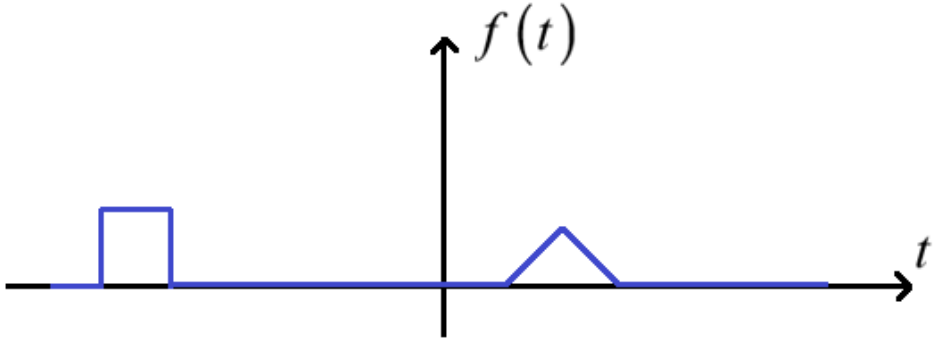
$$f_o(t) = \frac{1}{2}\left(f(t) - f(-t)\right) \quad \text{odd component}$$

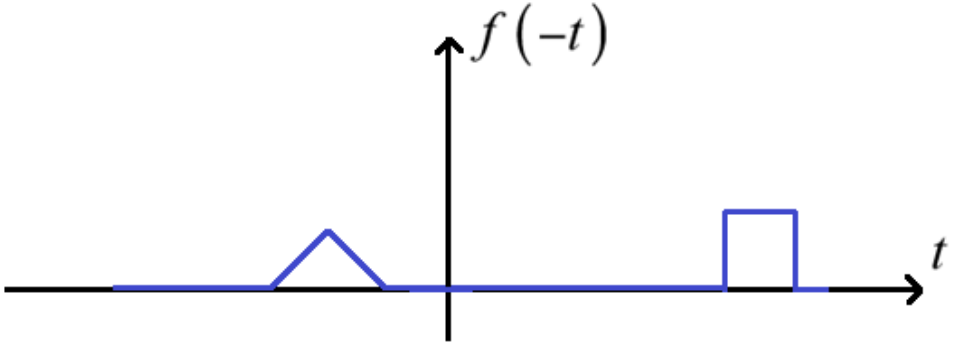
$$f(t) = f_e(t) + f_o(t)$$

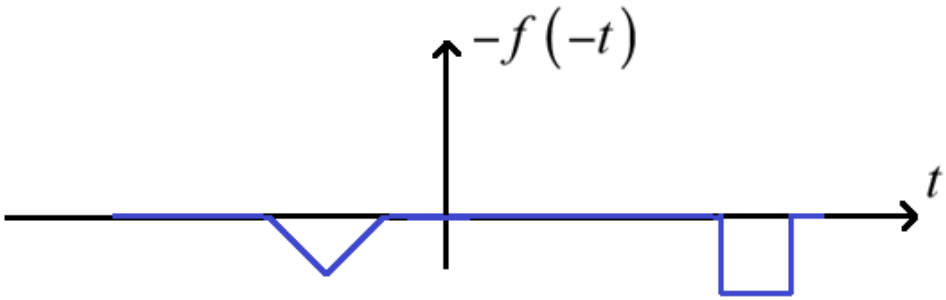
Ch1: Signals & Systems – Classification of signals

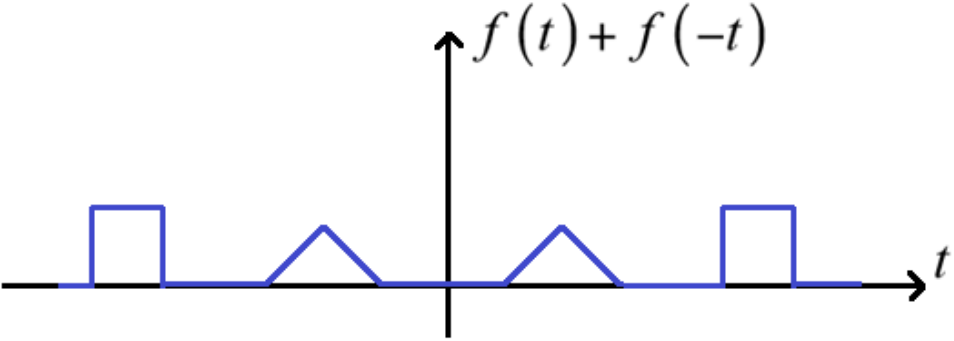
Decomposition in even and odd components

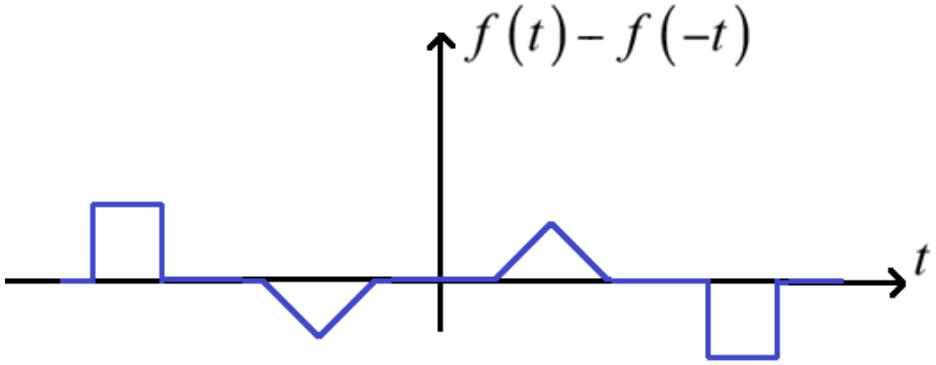




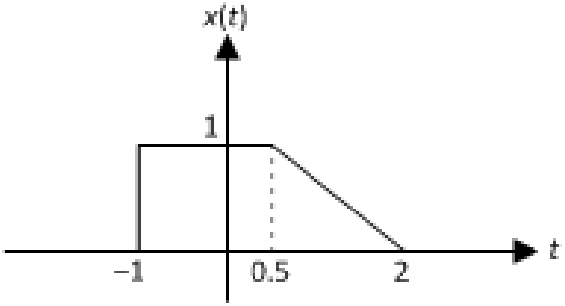




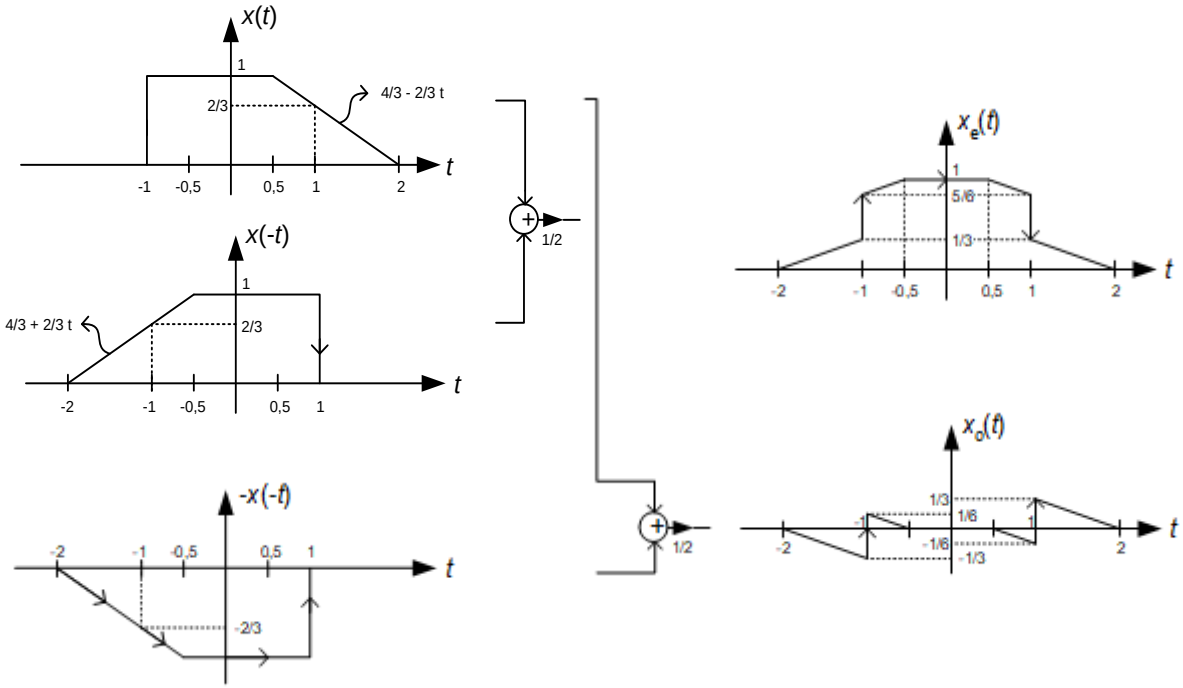




Ex: Obtain the graphics of EVEN and ODD parts of the $x(t)$ signal.



Sol: $x_e(t) = \frac{x(t)+x(-t)}{2}$, $x_o(t) = \frac{x(t)-x(-t)}{2}$



Ch1: Signals & Systems – Classification of signals

Energy and Power

- Signal energy

$$E_\infty \triangleq \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2\,dt = \int_{-\infty}^{+\infty}|x(t)|^2\,dt,$$

$$E_\infty \triangleq \lim_{N\to\infty}\sum_{n=-N}^{+N}|x[n]|^2 = \sum_{n=-\infty}^{+\infty}|x[n]|^2.$$

- Power
  - The power is the time average (mean) of the squared signal amplitude, that is the *mean-squared* value of x(t)

$$P_\infty \triangleq \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2\,dt$$

$$P_\infty \triangleq \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{+N}|x[n]|^2$$

Ch1: Signals & Systems – Classification of signals

Energy and Power

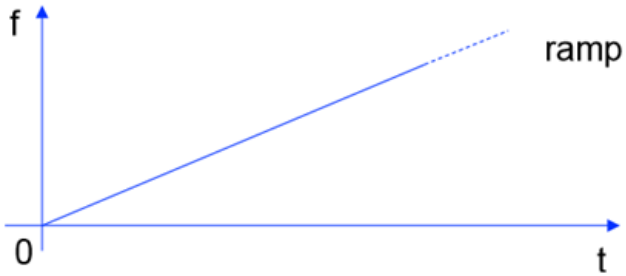
- The square root of the power is the root mean square (*rms*) value
  - This is a very important quantity as it is the most widespread measure of similarity/dissimilarity among signals
  - It is such that a constant signal whose amplitude is =rms holds the same power content of the signal itself
  - It is the basis for the definition of the Signal to Noise Ratio (SNR)

$$SNR = 20\log_{10}\left(\sqrt{\frac{P_{signal}}{P_{noise}}}\right)$$

Ch1: Signals & Systems – Classification of signals

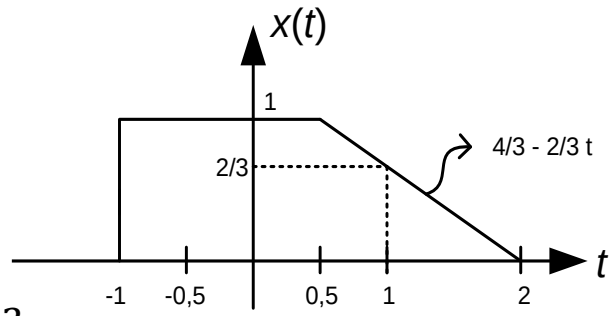
Energy and Power Signals

- A signal with finite energy is an energy signal
  - Necessary condition for a signal to be of energy type is that the amplitude goes to zero as the independent variable tends to infinity
- A signal with finite and different from zero power is a power signal
  - The mean of an entity averaged over an infinite interval exists if either the entity is periodic or it has some statistical regularity
  - A power signal has infinite energy and an energy signal has zero power
  - There exist signals that are neither power nor energy, such as the ramp
- All practical signals have finite energy and thus are energy signals
  - It is impossible to generate a real power signal because this would have infinite duration and infinite energy, which is not doable.
    - There exists signals for which neither the energy nor the power are finite



Ex: a) Calculate the energy of the $x(t)$ signal.

b) The $x(t)$ signal is periodized with the fundamental period T=4. Draw the periodic signal and calculate the mean value, mean power and rms value of the periodic signal.



Sol: a) $$E_x = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-1}^{2} x^2(t)dt = \int_{-1}^{0,5} 1^2 dt + \int_{0,5}^{2} \left(\frac{4}{3} - \frac{2}{3}t\right)^2 dt$$

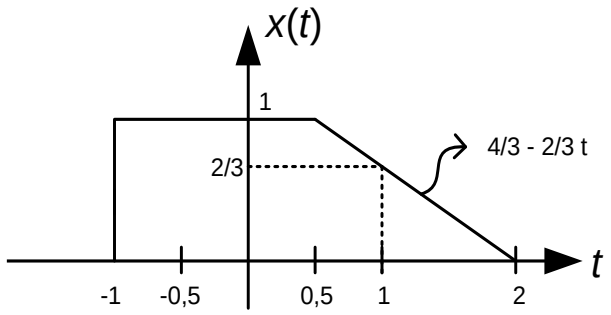
$$= 1{,}5 + \left(\frac{16}{9}t - \frac{8}{9}t^2 + \frac{4}{27}t^3\right)\Big|_{0,5}^{2}$$

$$= 2 \text{ joule}$$

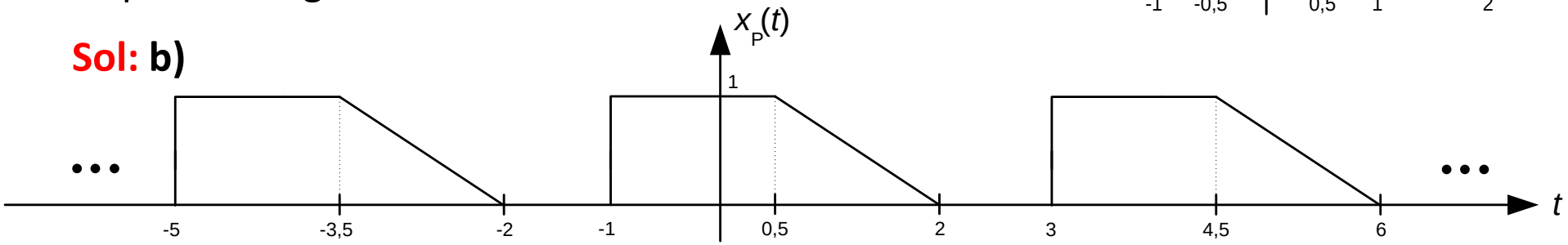
(It can be considered as the energy released on a 1 Ω resistance.)

Ex: a) Calculate the energy of the $x(t)$ signal.

b) The $x(t)$ signal is periodized with the fundamental period T=4. Draw the periodic signal and calculate the mean value, mean power and rms value of the periodic signal.



Sol: b)



$$x_{\text{mean}} = \frac{1}{T}\int_{-T/2}^{T/2} x_P(t)dt = \frac{1}{4}\left[\int_{-1}^{0,5} 1dt + \int_{0,5}^{2}\left(\frac{4}{3}-\frac{2}{3}t\right)dt\right] = \frac{1}{4}\left[\frac{3}{2} + \left(\frac{4}{3}t - \frac{t^2}{3}\right)\Big|_{0,5}^{2}\right] = \frac{9}{16}$$

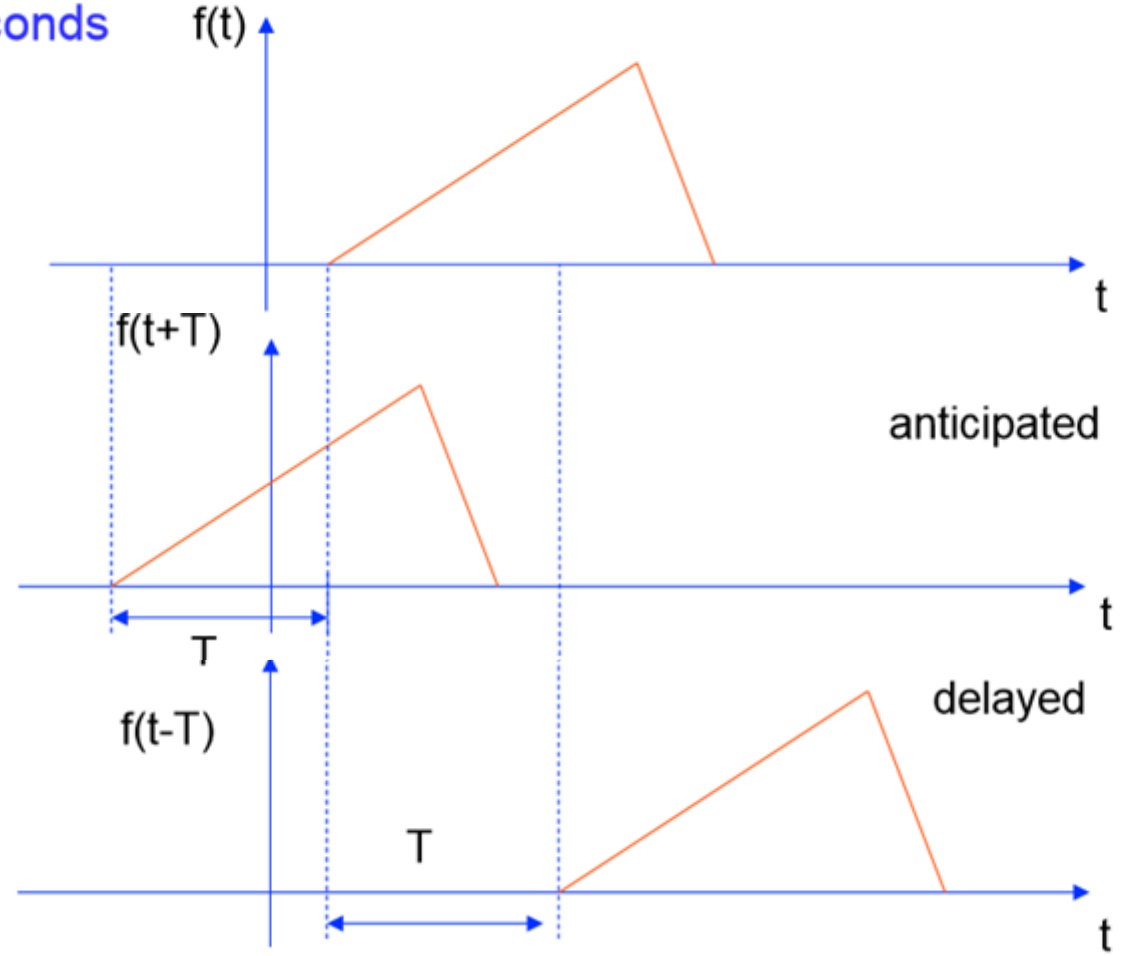
$$P_{\text{mean}} = \frac{1}{T}\int_{-T/2}^{T/2} x_P^2(t)dt = \frac{1}{4}\left[\int_{-1}^{0,5} 1^2dt + \int_{0,5}^{2}\left(\frac{4}{3}-\frac{2}{3}t\right)^2 dt\right] = \frac{1}{4}[2] = \frac{1}{2}\ \text{watt}$$

$$x_{\text{rms}} = \sqrt{P_{\text{mean}}} = \frac{1}{\sqrt{2}}$$

Ch1: Signals & Systems – Classification of signals

Some Useful Signal Operations – Shifting, Scaling, Inversion

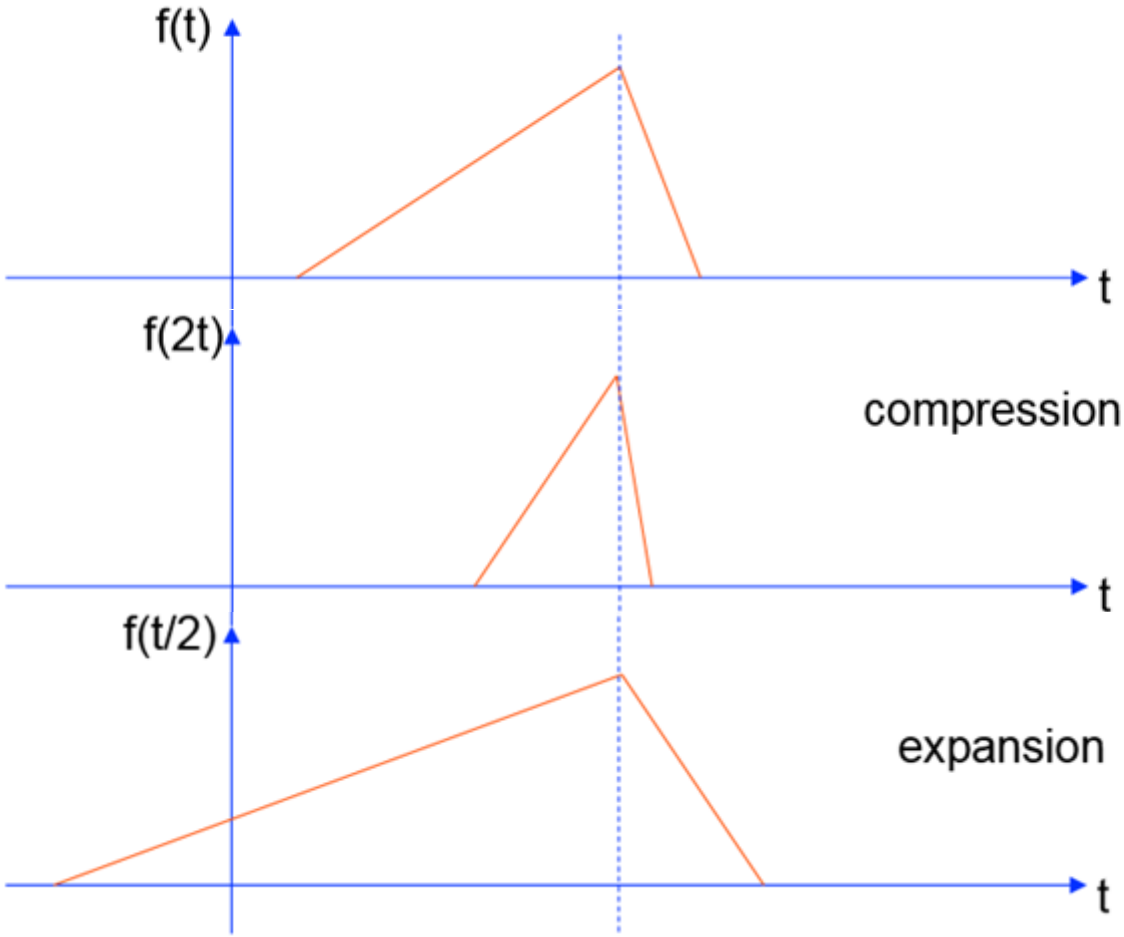
- **Shifting**: consider a signal f(t) and the same signal delayed/anticipated by T seconds



Ch1: Signals & Systems – Classification of signals

Some Useful Signal Operations – Shifting, Scaling, Inversion

- (Time) Scaling: compression or expansion of a signal in time



compression $\varphi(t) = f(2t)$

expansion $\varphi(t) = f(t/2)$

Ch1: Signals & Systems – Classification of signals

Some Useful Signal Operations – Shifting, Scaling, Inversion

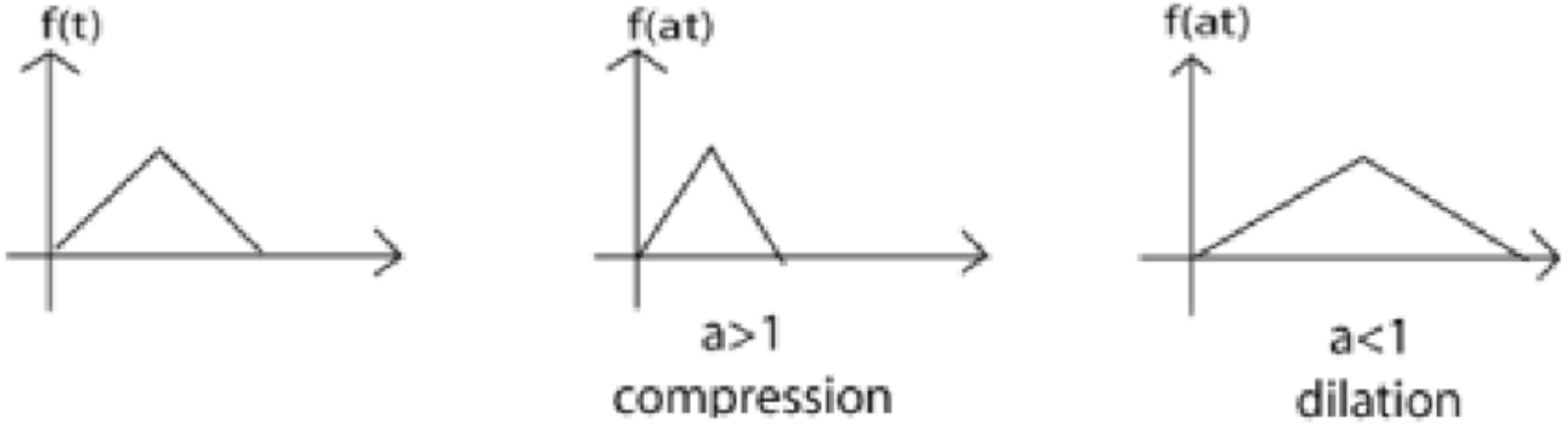
- Scaling: generalization

$$a > 1$$

$$\varphi(t) = f(at) \rightarrow \text{compressed version}$$

$$\varphi(t) = f\left(\frac{t}{a}\right) \rightarrow \text{dilated (or expanded) version}$$

Viceversa for $a < 1$

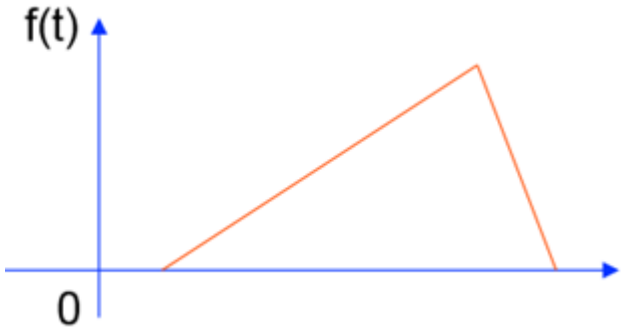


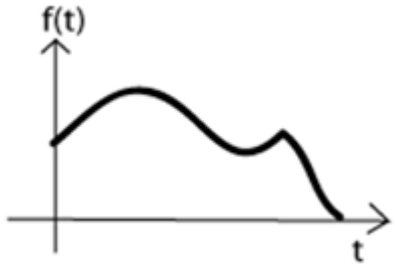
Ch1: Signals & Systems – Classification of signals

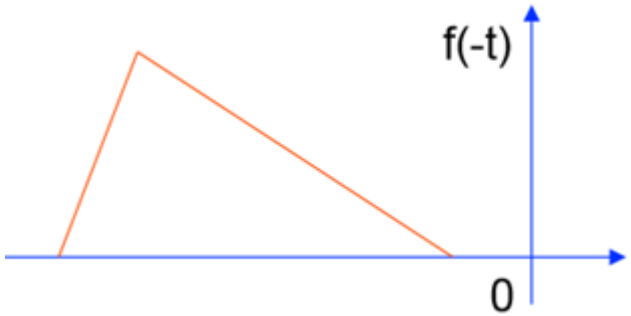
Some Useful Signal Operations – Shifting, Scaling, Inversion (Reversal)

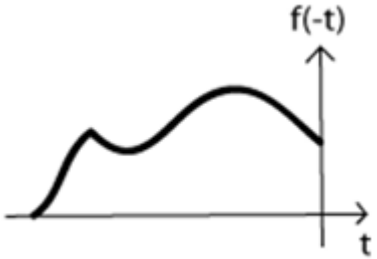
- (Time) inversion: mirror image of f(t) about the vertical axis

$$\varphi(t) = f(-t)$$

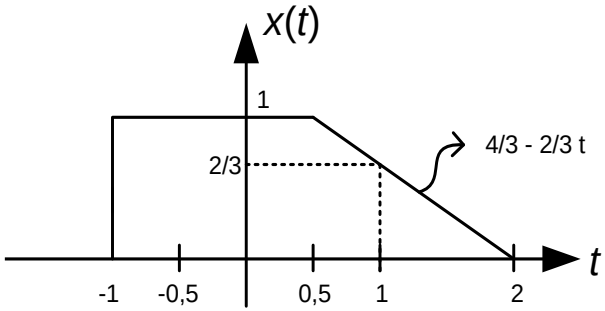




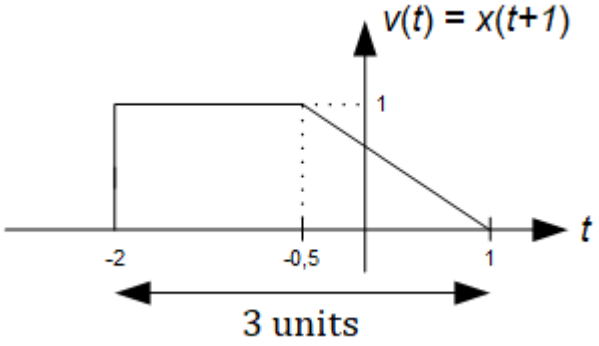


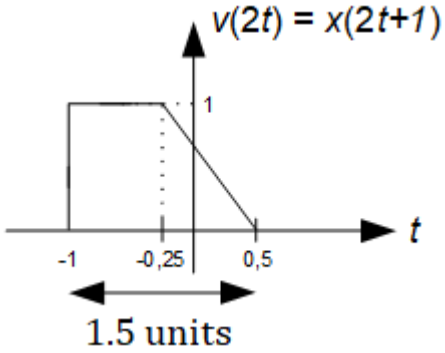


Ex: Draw $x(2t+1)$ and $x\left(4-\frac{3}{2}t\right)$ signals

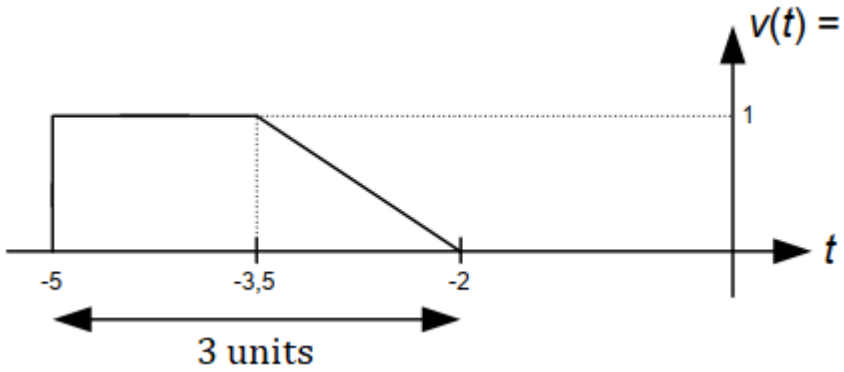


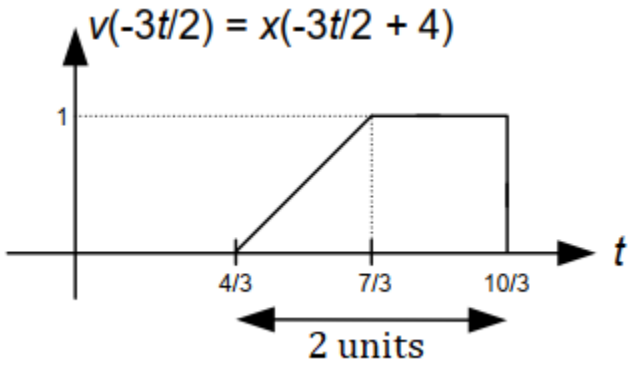
Sol: For $x(2t+1)$





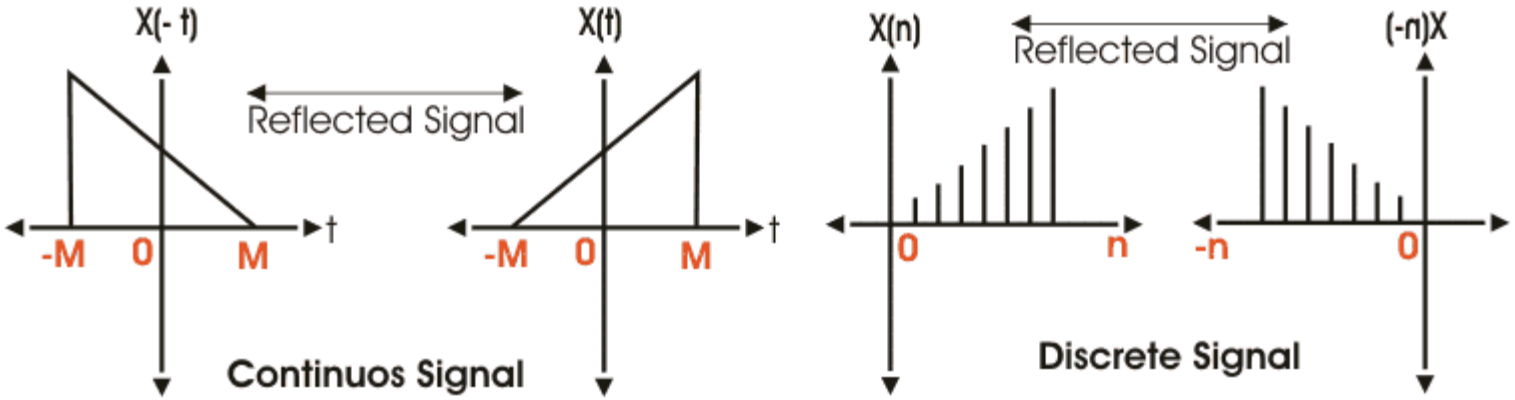
For $x\left(4-\frac{3}{2}t\right)$

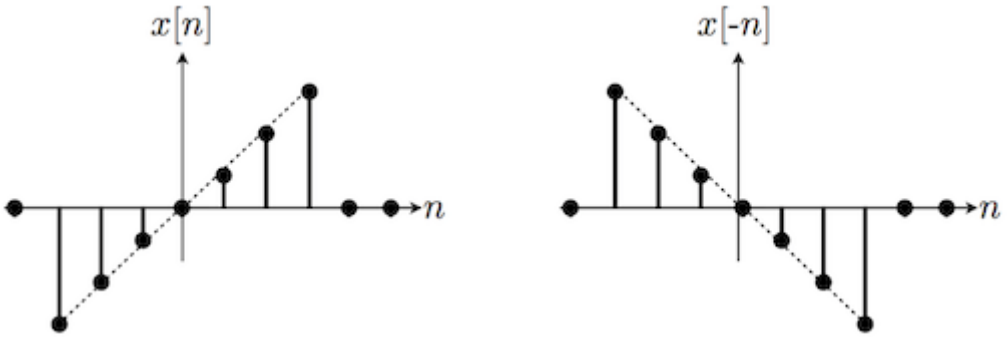




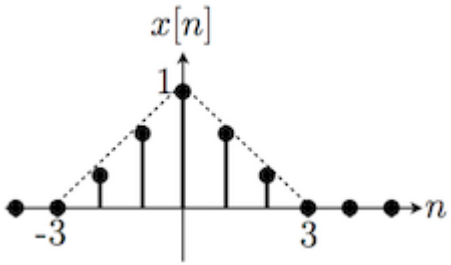
Ch1: Signals & Systems – Classification of signals

Ex: In Discrete-time (time inversion/reversal)



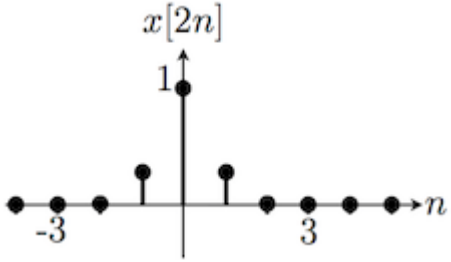


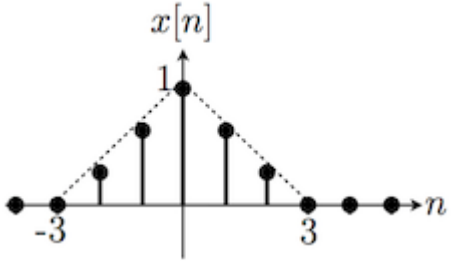
Ex: In Discrete-time (time scaling)



$x[2n]$

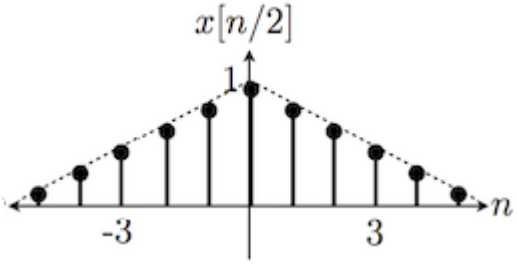
$(compression)$





$x[\frac{n}{2}]$

$(expansion)$



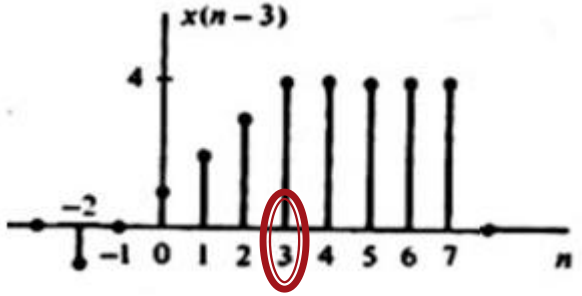
Ex: In Discrete-time (time scaling)



$x[n-3]$

$(n-3=0)$

$(n=3)$



(New origin of the signal)



$x[n+2]$

$(n+2=0)$

$(n=-2)$



(New origin of the signal)

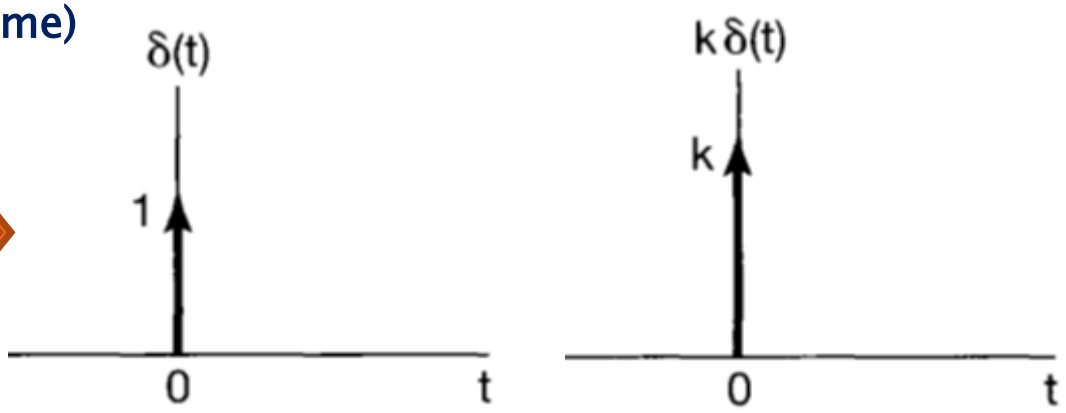
Ch1: Signals & Systems – Classification of signals

Some Useful Functions - UNIT IMPULSE & UNIT STEP

- Unit impulse function (continuous-time)

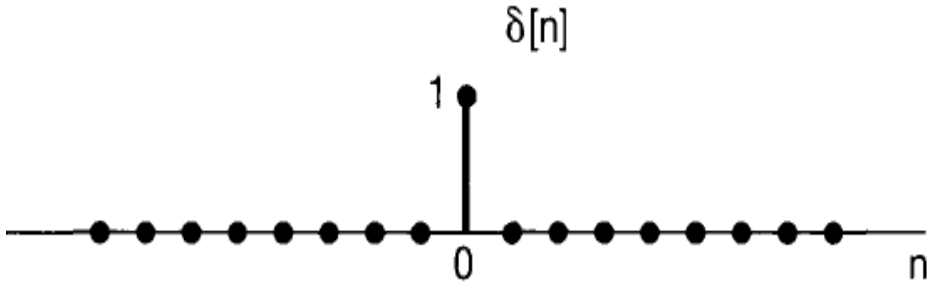
$$\delta(t) = 0 \quad t \neq 0$$

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$



- Unit impulse function (discrete-time)

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

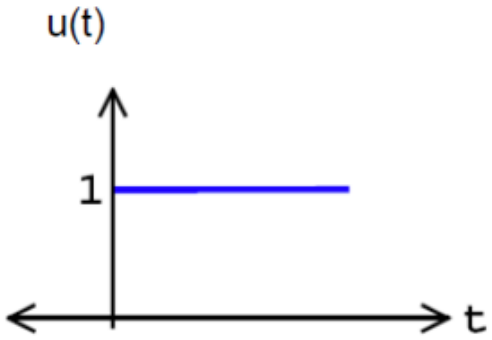


Ch1: Signals & Systems – Classification of signals

Some Useful Functions - UNIT IMPULSE & UNIT STEP

- Unit step function (continuous-time)

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases},$$

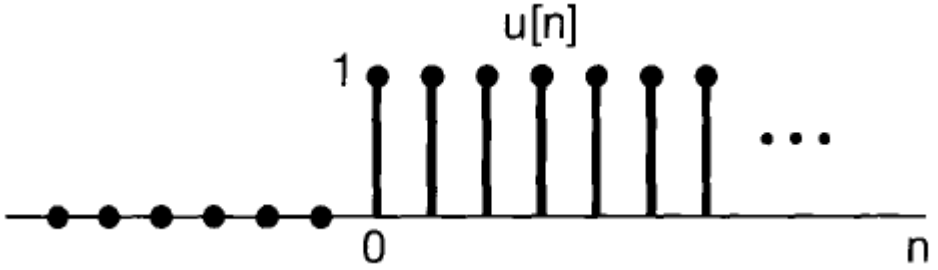


$$\delta(t) = \frac{du(t)}{dt}.$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau.$$

- Unit step function (discrete-time)

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}.$$



$$\delta[n] = u[n] - u[n-1].$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m].$$

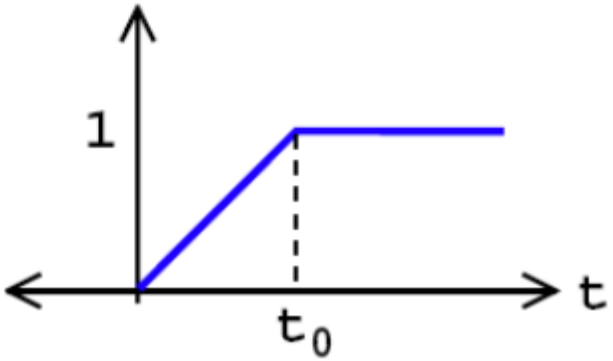
$$u[n] = \sum_{k=0}^{\infty} \delta[n-k].$$

Ch1: Signals & Systems – Classification of signals

Some Useful Functions - RAMP

- Ramp function (continuous time)

$$r(t) = \begin{cases} 0 \text{ if } t < 0 \\ \frac{t}{t_0} \text{ if } 0 \leq t \leq t_0 \\ 1 \text{ if } t > t_0 \end{cases}$$



Ch1: Signals & Systems – Classification of signals

Some Useful Functions

- Continuous-Time Complex Exponential $x(t) = Ae^{jwt}$
- Euler's Relations:
  - $Ae^{j\theta} = Acos(\theta) + jAsin(\theta)$
  - $cos(\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2}$, $\sin(\theta) = \frac{e^{j\theta}-e^{-j\theta}}{2j}$
    - $Ae^{jwt} = Acos(wt) + jAsin(wt)$
    - $cos(wt) = \frac{e^{jwt}+e^{-jwt}}{2}$, $\sin(\theta) = \frac{e^{jwt}-e^{-jwt}}{2j}$
- Discrete-Time Complex Exponential
- t=nT $x[n] = Ae^{jwnT} = Ae^{snT}$ $(s = jw)$

Ch1: Signals & Systems – Classification of signals

Some Useful Functions

- Continuous-Time Complex Exponential $x(t) = Ae^{jwt}$
- Generalization:
  - $s = \sigma + jw \quad \rightarrow\rightarrow\rightarrow \quad x(t) = e^{st}$
  - $x(t) = e^{st} = e^{(\sigma+jw)t} = e^{\sigma t}\, e^{jwt} = e^{\sigma t}\,(cos(wt) + jsin(wt))$
  - For s^*(the conjugate of s) $\rightarrow\rightarrow\rightarrow s^* = \sigma - jw$
  - $e^{s^*t} = e^{(\sigma-jw)t} = e^{\sigma t}\, e^{-jwt} = e^{\sigma t}\,(cos(wt) - jsin(wt))$
    - $e^{\sigma t}\big(cos(wt)\big) = \frac{1}{2}\big(e^{st} + e^{s^*t}\big)$

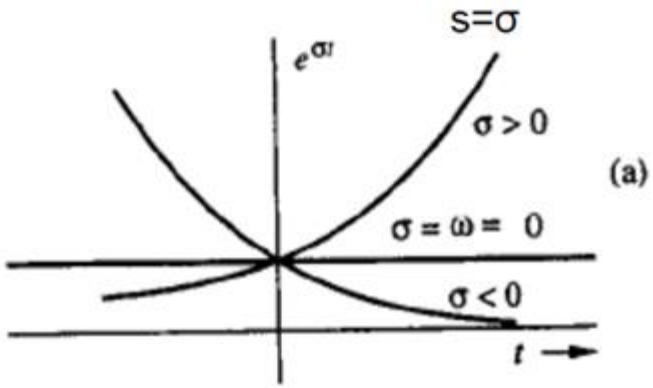
Ch1: Signals & Systems – Classification of signals

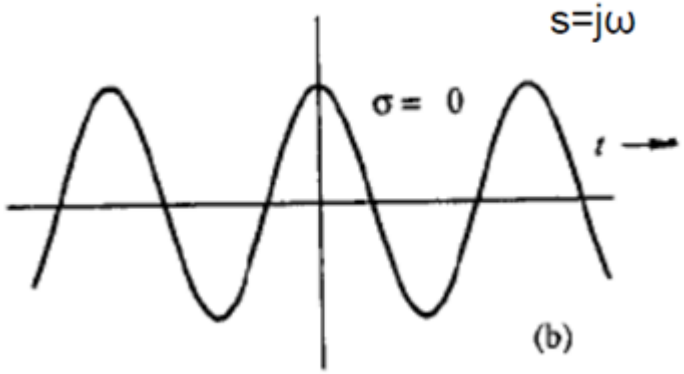
Some Useful Functions

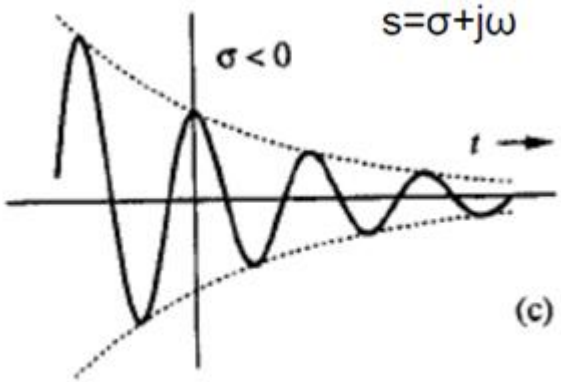
$$x(t) = e^{st}$$

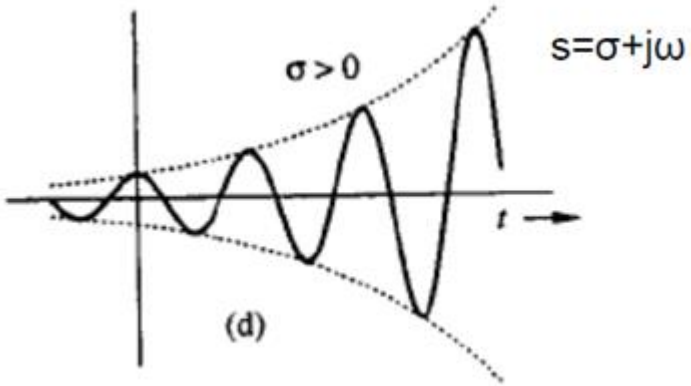
$$s = \sigma + jw$$

The exponential function









Ch1: Signals & Systems – Classification of signals

Some Useful Functions

READING ASSIGNMENT

Continuous-Time Complex Exponential and Sinusoidal Signals !!!

Continuous-Time Complex Exponential and Sinusoidal Signals !!!

Periodicity Properties of Discrete-Time Complex Exponentials !!!

Comparison of the signals $e^{j\omega_0 t}$ and $e^{j\omega_0 n}$.

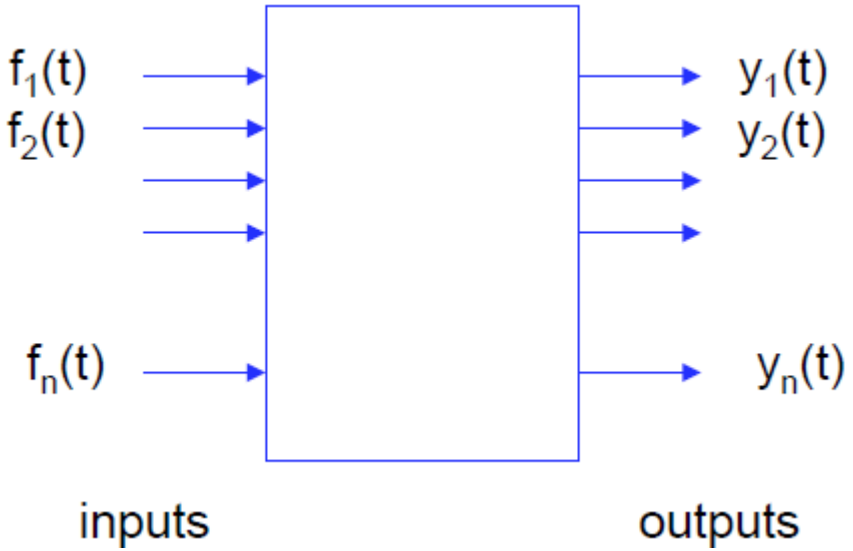
Ch1: Signals & Systems – Systems

System

- A system can be viewed as any process that results in the transformations of signals.

- Physical systems in the broadest sense are an interconnection of
  - components,
  - devices
  - and subsystems.

- A system is characterized by
  - inputs
  - outputs
  - Rules of operation (mathematical model of the system)



Ch1: Signals & Systems – Systems

System

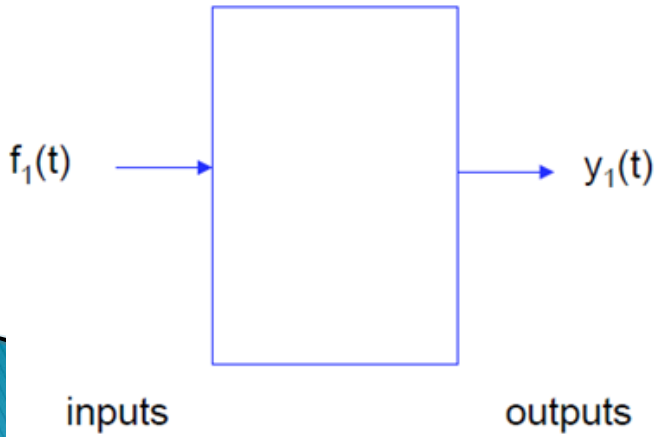
- Study of systems : **Mathematical modeling**, **analysis**, **design**
  - **ANALYSIS:** How to **determine the system output** given the input and the system mathematical model.

$$Input + System \rightarrow Output$$

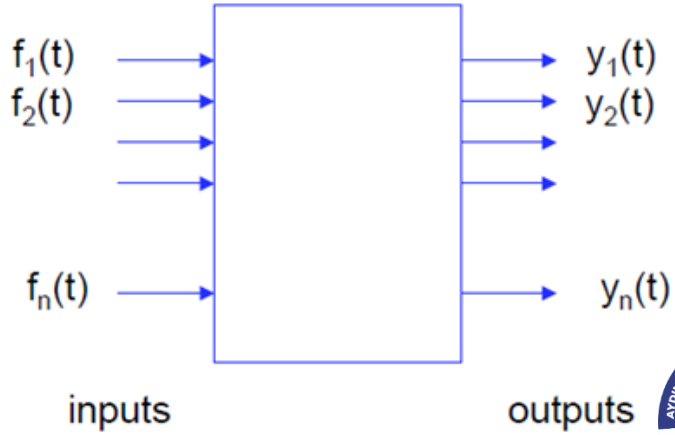
  - **SYNTHESIS (or design):** How to **design a system** that will produce the desired set of outputs for given inputs.

$$Input + Output \rightarrow System$$

- SISO: single input single output -



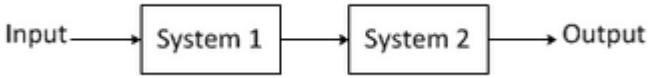
MIMO: multiple input multiple output



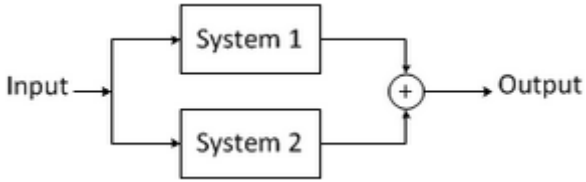
Ch1: Signals & Systems – Systems

Interconnection of Sytems

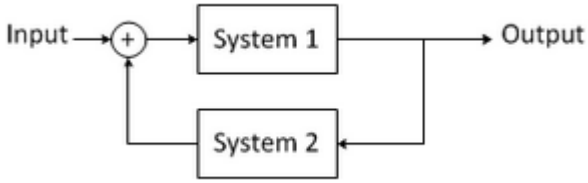
- **Serial or Cascade**



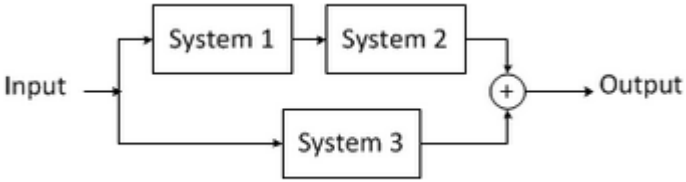
- **Parallel**



- **Feedback**



- **Ex:**



Ch1: Signals & Systems – System Properties

Sytem Properties

- Memory
- Invertibility
- Causality (Nonanticipative)
- Stability (Bounded Input, Bounded Output - BIBO)
- Time Invariance
- Linearity

- **For a system to passes a given property, the property must hold for every possible input signals.**

- **A counter example is sufficient to disprove the property.**

Ch1: Signals & Systems – System Properties

Memory

- A system is said to be ***memoryless*** if its output for each value of the independent variable at a given time is dependent on the input at only that same time.

Ex:

$y(t) = x(t)$, $\quad y[n] = x[n]$

$y(t) = \alpha x(t)$, $\quad y[n] = \beta x[n]$

$y(t) = (x(t) + x^2(t))^3$,

O/p depends present value of i/p
Hence, these Systems are memoryless

$y[n] = x[n-1]$, $\quad y[n] = x[n-100]$, $\quad y[n] = x[n+2]$

$y(t) = x(t-6)$, $\quad y(t) = 2x(t+1) + x(t)$

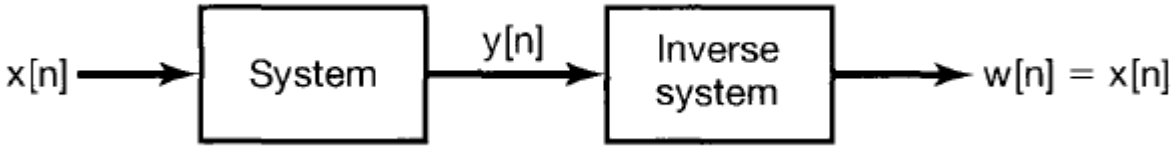
$$y[n] = \sum_{k=-\infty}^{n} x[k],$$

O/p depends past and/or future value of i/p
Hence, these Systems have memory

Ch1: Signals & Systems – System Properties

Invertibility

- A system is said to be ***invertible*** if distinct inputs lead to distinct outputs.
- Or a system is said to be ***invertible*** if we can obtain its input by observing its output.



Ex:

$$y(t) = 2x(t), \quad \text{y[n]} = 3\text{x[n]} \longrightarrow \text{x}(t) = \frac{y(t)}{2}, \quad \text{x[n]} = \frac{y[n]}{3}$$

O/p is obtainable by observing (or using) i/p
Hence, these Systems are invertible

$$y(t) = (x(t))^2, \longrightarrow \text{x}(t) = \sqrt{y(t)},$$

NO !!!
Assume x(t) = (-2), can you obtain x(t) back?

Hence, this system is NOT invertible

Ch1: Signals & Systems – System Properties

Causality

- A system is ***causal*** if the output at any time depends on values of the input at only the present and/or past times. Such a system is often referred to as being ***nonanticipative***, as the system output does not anticipate future values of the input.

Ex:

$y(t) = x(t)$, $\quad y[n] = x[n]$ → O/p depends present and/or past value of i/p

Hence, these Systems are causal

$y(t) = x(t-6)$, $\quad y(t) = 2x(t) - 3x(t-1)$

$y[n] = x[n] + x[n-1] - 3x[n-4]$

$y[n] = x[n+2]$, $\quad y(t) = 5x(t) - 2x(t+1)$, → O/p depends future value of i/p

Hence, these Systems are NOT causal.

$$y[n] = \frac{1}{2M+1} \sum_{k=-M}^{+M} x[n-k].$$

$y(t) = (t+1)x(t-6)$, This is a causal system, (t+1) is just a coefficient !

Ch1: Signals & Systems – System Properties

Stability (BIBO)

- A ***stable*** system is one in which small inputs lead to responses that do not diverge.
- **B**ounded **I**nput must reason for a **B**ounded **O**utput

$$If\ |x(t)| < \infty \ \rightarrow\ |y(t)| < \infty$$
$$If\ |x[n]| < \infty \ \rightarrow\ |y[n]| < \infty$$

Ex:

$$y[n] = \sum_{k=-\infty}^{n} u[k] = (n+1)u[n].$$

That is, $y[0] = 1$, $y[1] = 2$, $y[2] = 3$, and so on, and $y[n]$ grows without bound.

Hence, this system is NOT (BIBO) stable.

Ch1: Signals & Systems – System Properties

Time Invariance

- A system is ***time invariant (TI)*** if the behavior and characteristics of the system are fixed over time. Or, a system is ***TI*** if a time shift in the input signal causes the **same** amount of time shift in the output signal.

$$IF\ x(t) \rightarrow y(t) \quad THEN\ x(t-t_0) \rightarrow y(t-t_0)$$

time invariant (TI)

$$IF\ x[n] \rightarrow y[n] \quad THEN\ x[n-n_0] \rightarrow y[n-n_0]$$

Ex:

$$y(t) = \sin\left[x(t)\right].$$

$$y_1(t) = \sin\left[x_1(t)\right] \Rightarrow x_2(t) = x_1(t-t_0). \Rightarrow y_2(t) = \sin\left[x_2(t)\right] = \sin\left[x_1(t-t_0)\right].$$

$$y_1(t-t_0) = \sin\left[x_1(t-t_0)\right]. \Rightarrow y_2(t) = y_1(t-t_0),$$

Hence, this system is TI

Ch1: Signals & Systems – System Properties

Time Invariance

- A system is ***time invariant (TI)*** if the behavior and characteristics of the system are fixed over time. Or, a system is ***TI*** if a time shift in the input signal causes the **same** amount of time shift in the output signal.

Ex: $y[n] = nx[n]$

$Lets\ define\ x_1[n] = x[n - n_0] \Rightarrow y_1[n] = n(x_1[n]) \Rightarrow y_1[n] = n(x[n - n_0])$

$Lets\ find\ y[n - n_0] \Rightarrow just\ write\ (n - n_0)\ instead\ of\ n\ in\ y[n] = nx[n]$

$$y[n - n_0] = (n - n_0)x[n - n_0]$$

$$y[n - n_0] = (n - n_0)x[n - n_0] \neq y_1[n] = n(x[n - n_0])$$

Hence, this system is NOT TI, which is TV

Ch1: Signals & Systems – System Properties

Linearity

- A ***linear*** system is a system that possesses the important property of superposition: If an input consists of the weighted sum of several signals, then the output is the superposition–that is, the weighted sum–of the responses of the system to each of those signals.

  - ❖ additivity property
  - ❖ scaling or homogeneity property

$$IF \quad x_1(t) \rightarrow y_1(t) \quad \& \quad x_2(t) \rightarrow y_2(t)$$

$$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t) = ay_1(t) + by_2(t)$$

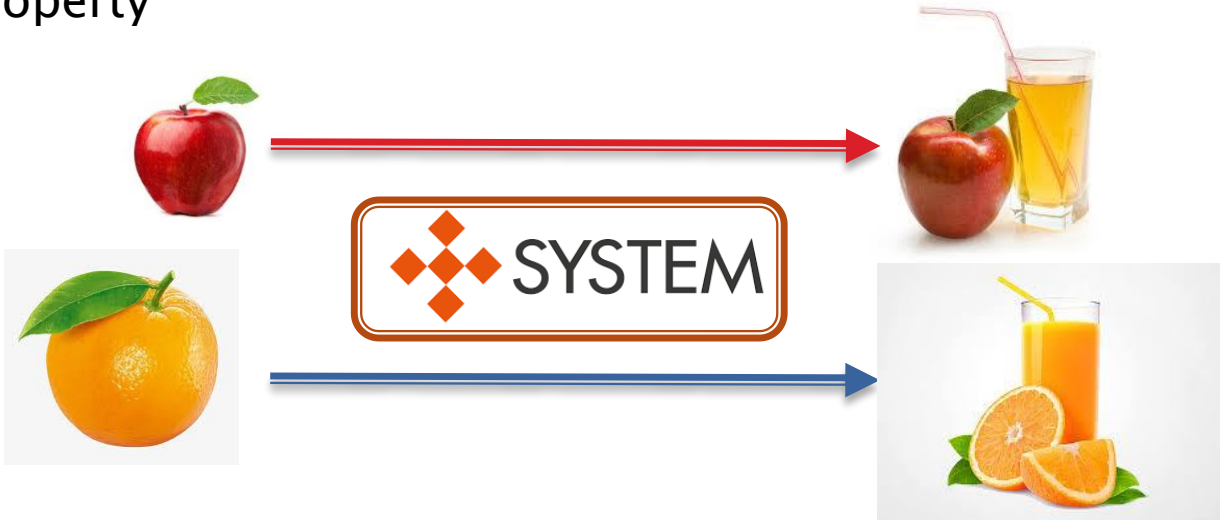
Then, such system is LINEAR…

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Linearity

- If an input consists of the weighted sum of several signals, then the output is the superposition–that is, the weighted sum–of the responses of the system to each of those signals.

- additivity property



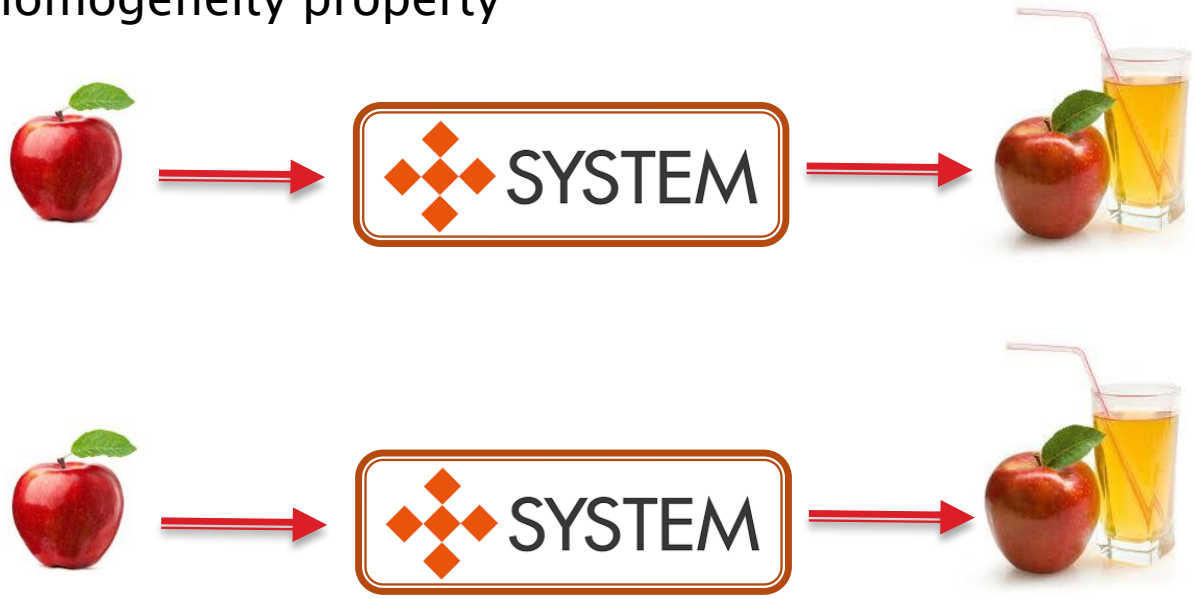


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- If an input consists of the weighted sum of several signals, then the output is the superposition–that is, the weighted sum–of the responses of the system to each of those signals.

- scaling or homogeneity property



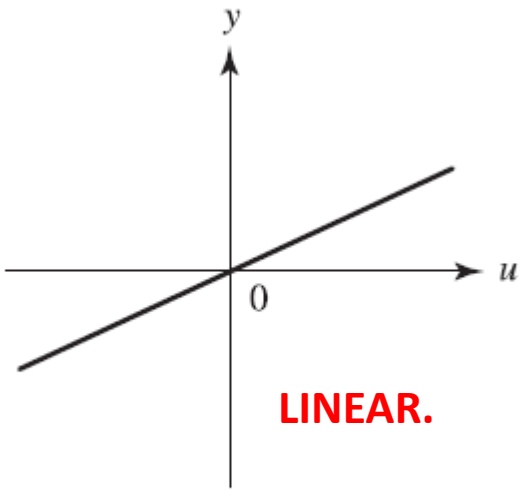
Ch1: Signals & Systems – System Properties

Linearity

Ex:

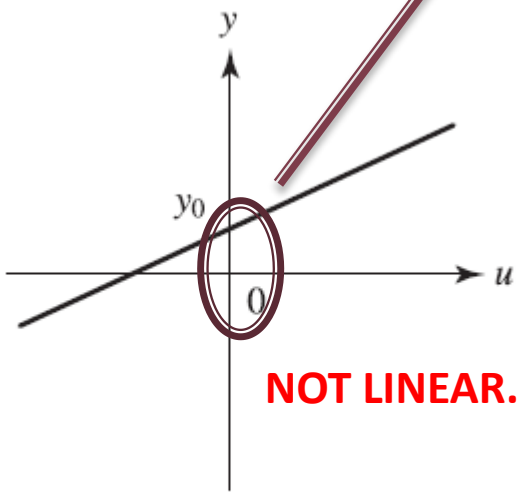
When u=0; y≠0

u denotes the input and y the output. Which of them is a linear system? Is it possible to introduce a new output so that the system in (b) is linear?



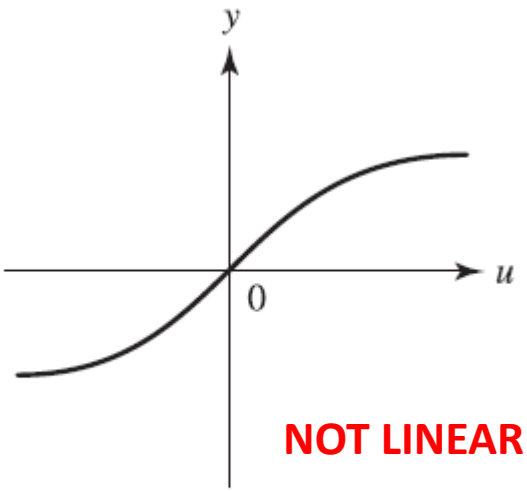
(a)

LINEAR.



(b)

NOT LINEAR.



(c)

NOT LINEAR.

For linear systems, zero input must respond to zero output !!!!

If we define a new output as $y_1(t) = y(t) - y_0$ then new system is LINEAR

Ex: Determine the properties of the system whose input-output relationships are given as follows.

(Is it linear, time invariant, causal, stable (BIBO), memoryless or not?)

$$y(t) = x(t)\cos(\omega_0 t)$$

Sol: **(i) Linearity**:

$$x_1(t) \rightarrow y_1(t) = x_1(t)\cos\omega_0 t$$

$$x_2(t) \rightarrow y_2(t) = x_2(t)\cos\omega_0 t$$

$$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t) = x_3(t)\cos\omega_0 t$$

$$= \big(ax_1(t) + bx_2(t)\big)\cos\omega_0 t$$

$$= \underbrace{ax_1(t)\cos\omega_0 t}_{y_1(t)} + \underbrace{bx_2(t)\cos\omega_0 t}_{y_2(t)}$$

$$= ay_1(t) + by_2(t)$$

The System is Linear

Ex: Determine the properties of the system whose input-output relationships are given as follows.

(Is it linear, time invariant, causal, stable (BIBO), memoryless or not?)

$$y(t) = x(t)\cos(\omega_0 t)$$

(ii) Time Invariance:

$$x(t) \rightarrow y(t)$$

$$x(t-t_0) \rightarrow y(t-t_0) \qquad ???$$

$$x(t) \rightarrow y(t) = x(t)\cos\omega_0 t$$

$$x(t-t_0) \rightarrow y'(t) = x(t-t_0)\cos\omega_0 t$$

$$y'(t) \neq y(t-t_0)$$

The System is not Time Invariant (It is time varying)

Ex: Determine the properties of the system whose input-output relationships are given as follows.

(Is it linear, time invariant, causal, stable (BIBO), memoryless or not?)

$$y(t) = x(t)\cos(\omega_0 t)$$

(iii) Causality:

The output at any time depends on values of the input at only the present time.

The System is causal

In this example the output at any time depends on values of the input at only the present time. The system would be causal if the output would depend also past times.

Ex: Determine the properties of the system whose input-output relationships are given as follows.

(Is it linear, time invariant, causal, stable (BIBO), memoryless or not?)

$$y(t) = x(t)\cos(\omega_0 t)$$

(iv) BIBO Stability:

$$|y(t)| = |x(t)\cos\omega_0 t| \le |x(t)|\underbrace{|cos\,\omega_0 t|}_{1} \le |x(t)|$$

The amplitude of the output is limited by the amplitude of the input.

The System is BIBO Stable

Ex: Determine the properties of the system whose input-output relationships are given as follows.

(Is it linear, time invariant, causal, stable (BIBO), memoryless or not?)

$$y(t) = x(t)\cos(\omega_0 t)$$

(v) Memory:

The output at any time depends on values of the input at only the present time.

The System is memoryless

END OF CHAPTER 1

Dr. Yılmaz KALKAN