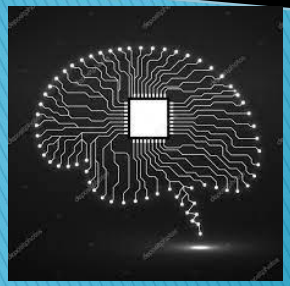
# AYDIN
# ADNAN MENDERES UNIVERSITY
# FACULTY OF ENGINEERING
## Department of Electrical and Electronics Engineering

# EE201 – Circuit Theory I

Prof. Dr. Yılmaz KALKAN

circuit theory






1. **Basic Concepts (Chapter 1 of Nilsson – Week 1)**

   Introduction, Current and Voltage, Power and Energy

2. **Basic Laws (Chapter 2&3 of Nilsson – Week 2)**

   Voltage and Current Sources, Ohm's Law,

   Kirchhoff's Laws, Resistors in parallel and in series, Voltage and Current Division

3. **Techniques of Circuit Analysis (Chapter 4 of Nilsson – Weeks 3-6)**

   Node Analysis, Node-Voltage Method and Dependent Sources, Mesh Analysis, Mesh-Current Method and Dependent Sources, Source Transformations, Thevenin and Norton Equivalents, Maximum Power Transfer, Superposition Theorem

   MIDTERM (Weeks 7&8)

4. **Operational Amplifier (Chapter 5 of Nilsson – Weeks 9-10)**

   Op-Amp Terminals & Ideal Op-Amp, Basic Op-Amp Circuits, Buffer circuit, Inverting and Non-inverting Amplifiers, Summing Inverter, Difference Amplifier, Cascade Op-Amp Circuits

5. **Capacitors and Inductors (Chapter 6 of Nilsson – Weeks 11)**

   Inductors, Capacitors, Series and Parallel Combinations of them

6. **First Order Circuits (Chapter 7 of Nilsson – Weeks 12 & 13)**

   The Natural Response of an RL & RC Circuits, The Step Response of RL and RC Circuits, A General Solution for Step and Natural Responses, Integrating Amplifier Circuit

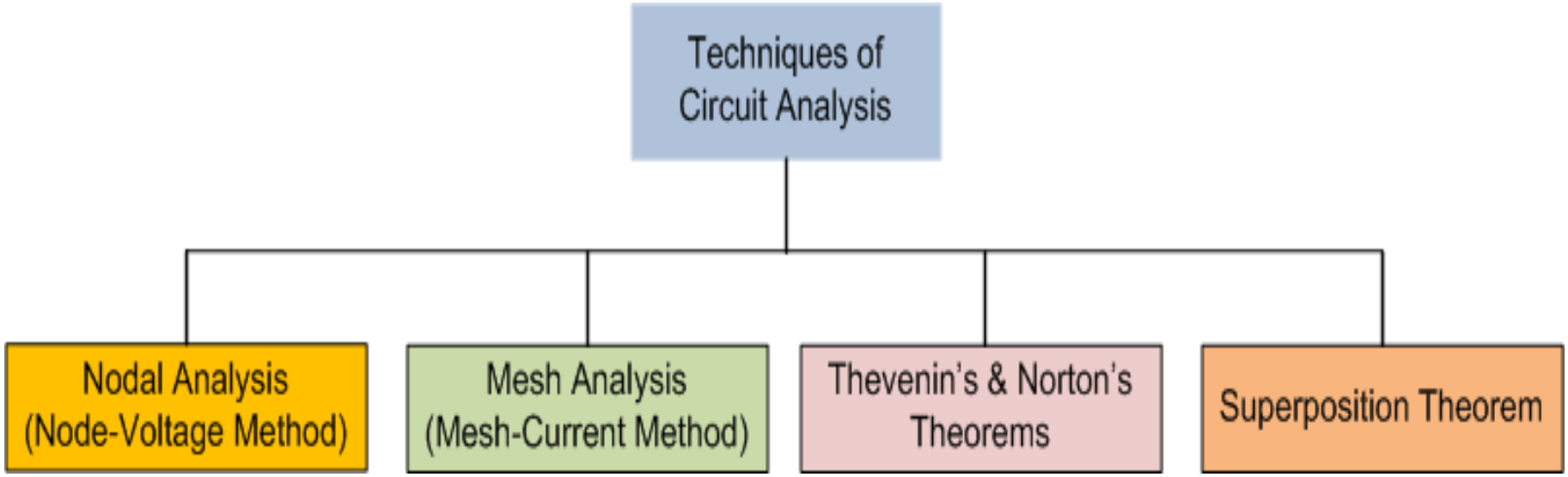
7. **Second Order Circuits (Chapter 8 of Nilsson – Weeks 14 & 15)**

   The Natural Response of a Parallel RLC Circuit, The Forms of Natural Response of a Parallel RLC Circuit, The Step Response of a Parallel RLC Circuit, Natural and Step Responses of a Series RLC Circuit



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# Ch3: Techniques of Circuit Analysis




- **Source Transformations**
- **Maximum Power Transfer**

## ❑ Nodal Analysis & Mesh Analysis

❑ **A major advantage** of analyzing circuits using Kirchhoff's laws is that we can analyze a circuit without tampering (changing) with its original configuration.

❑ **A major disadvantage** of this approach is that, for a large, complex circuit, tedious computation is involved.

❑ The growth in areas of application of electric circuits has led to an evolution from simple to complex circuits. To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis.

- ❑ **Source transformations**
- ❑ **Thevenin's Theorem**
- ❑ **Norton's Theorem**

## ❑ Source Transformations

❑ We have noticed that series-parallel combination and wye-delta transformation help to **simplify circuits**.

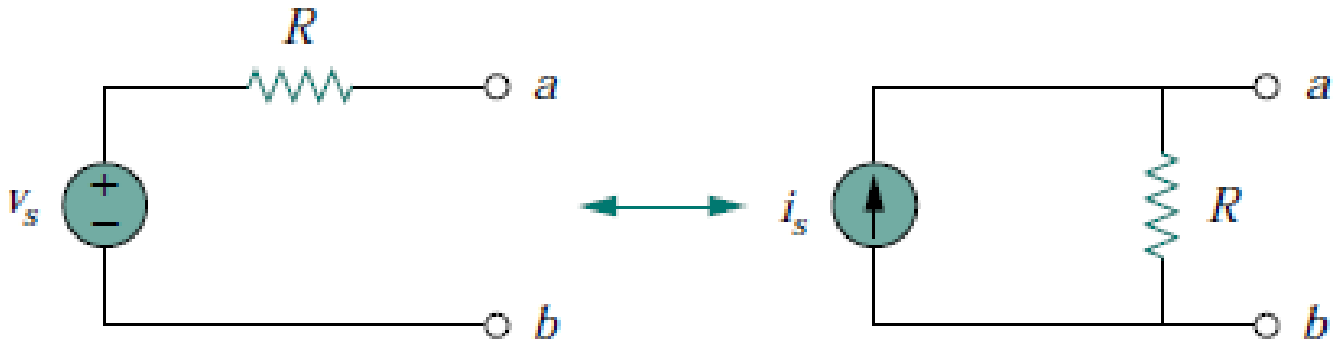
❑ Source transformation is another tool **for simplifying circuits**.

❑ Basic to these tools is the concept of **equivalence**.

**A source transformation is the process of replacing a voltage source $v_s$ in series with a resistor R by a current source is in parallel with a resistor R, or vice versa.**

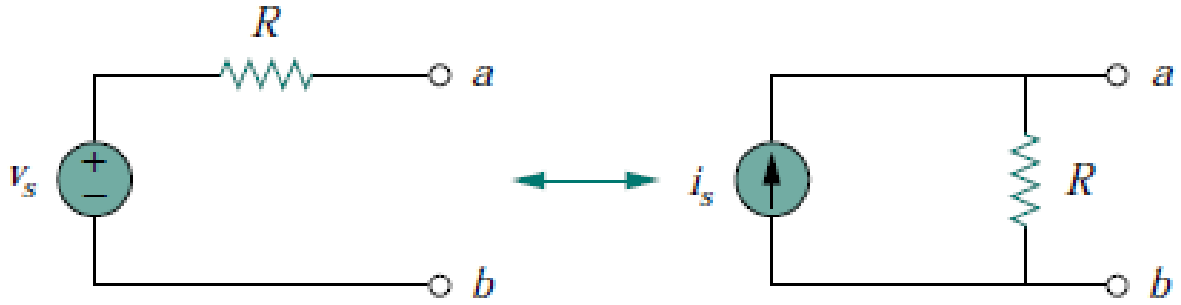
## ❑ Source Transformations

❑ A source transformation is the process of replacing a voltage source $v_s$ in series with a resistor R by a current source is in parallel with a resistor R, or vice versa.




❑ The two circuits are equivalent provided they have the same voltage-current relation at terminals *a-b.*

## ❑ Source Transformations




- ❑ Turned off the sources.
  - ❑ Voltage source is short circuited. (V=0)
  - ❑ Current source is open circuited. (i=0)

$$R_{ab} = R$$

- ❑ Short circuited terminals a-b.

$$i_{sc} = \frac{v_s}{R} \quad \& \quad i_{sc} = i_s$$

- ❑ In order for the two circuit s to be equivalent

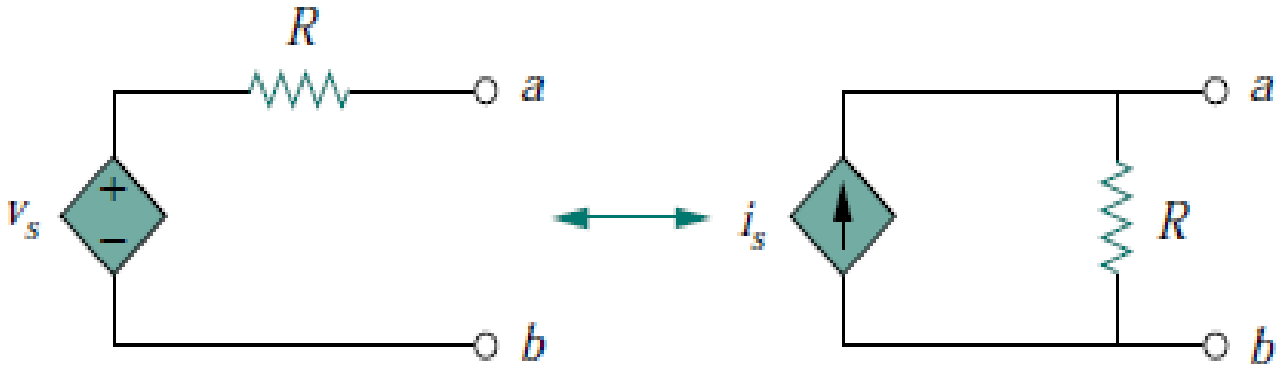
$$i_{sc} = i_s = \frac{v_s}{R}$$

**Hence, source transformation requires that:**

$$i_s = \frac{v_s}{R} \quad or \quad v_s = i_s R$$

## ❑ Source Transformations

❑ Source transformation also applies to dependent sources, provided we carefully handle the dependent variable

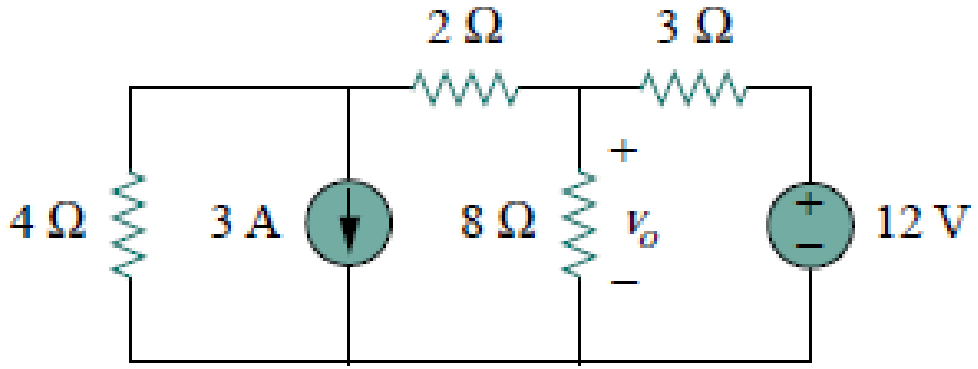


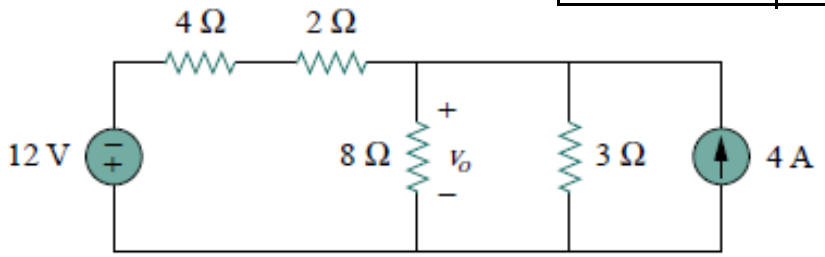

❑ Like the wye-delta transformation, a source transformation does not affect the remaining part of the circuit.

❑ If **R=0** or **R=∞**, source transformation is **NOT POSSIBLE**.

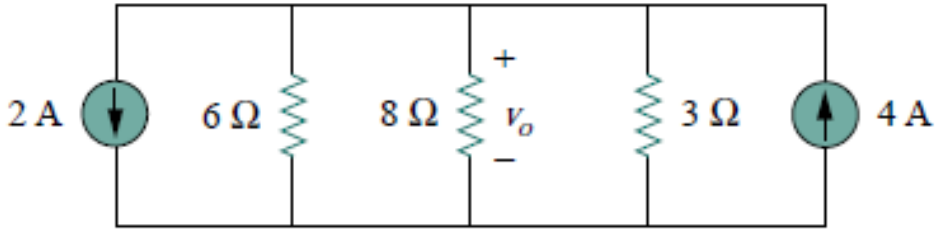
## ❑ Source Transformations

**Example:** Use source transformation to find $v_0$ *in the circuit in Figure below?*

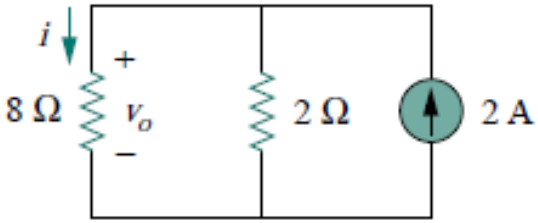



(a)




(b)



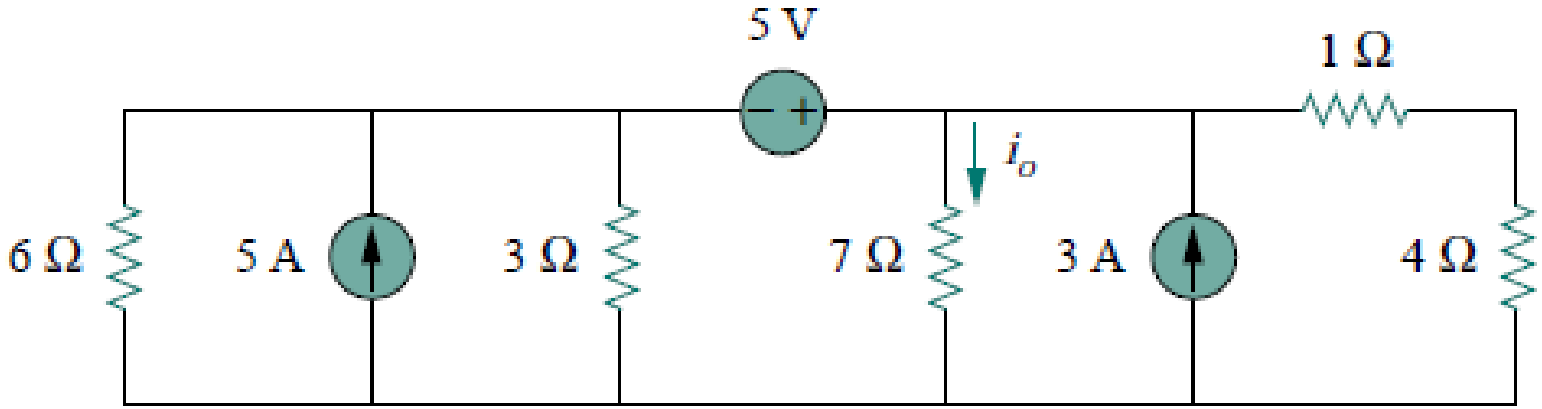

(c)

$$i_s = \frac{v_s}{R} \quad or \quad v_s = i_s R$$

$$v_0 = 3.2V$$

## ❑ Source Transformations

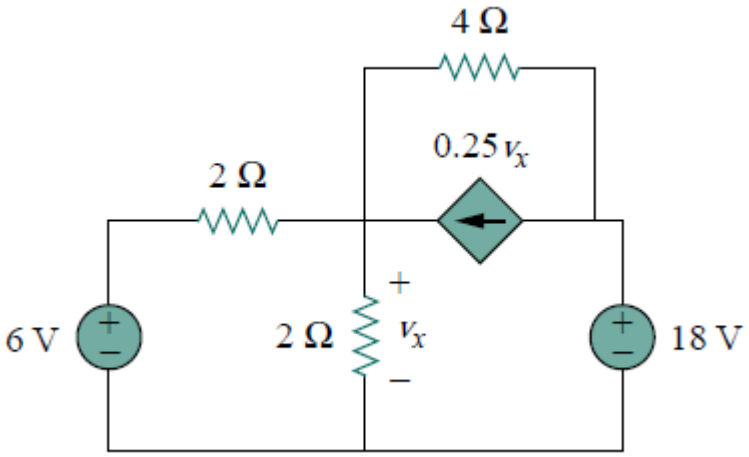
**Practice Problem:** Use source transformation to find $i_0$ *in the circuit in Figure below?*




$$i_0 = 1.78A$$

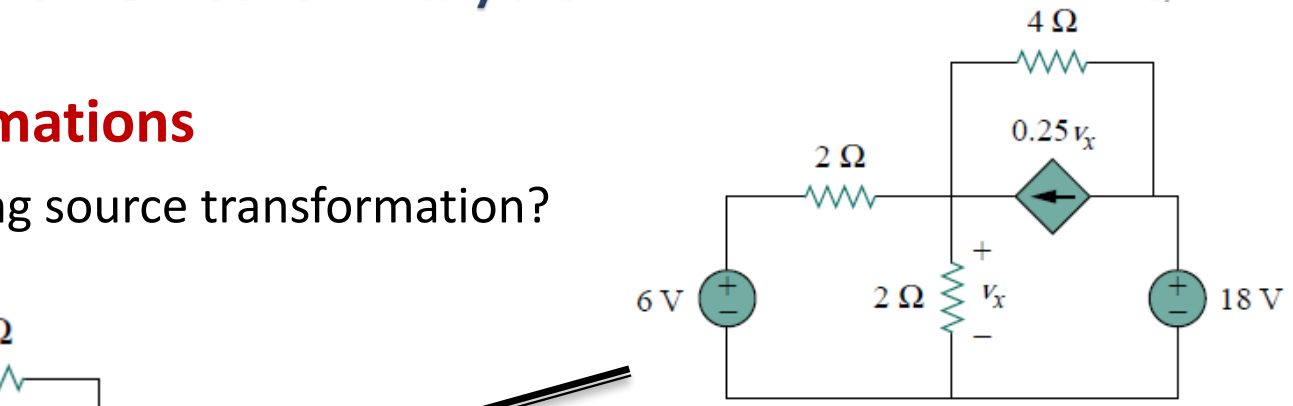
## ❑ Source Transformations

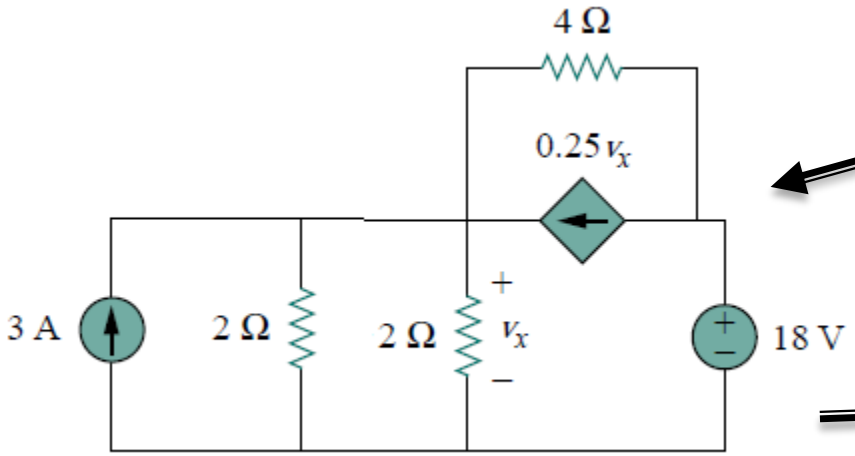
**Example:** Find $v_x$ using source transformation?

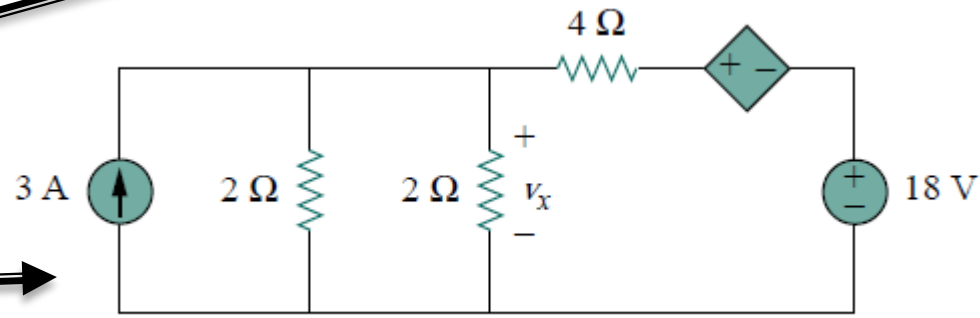


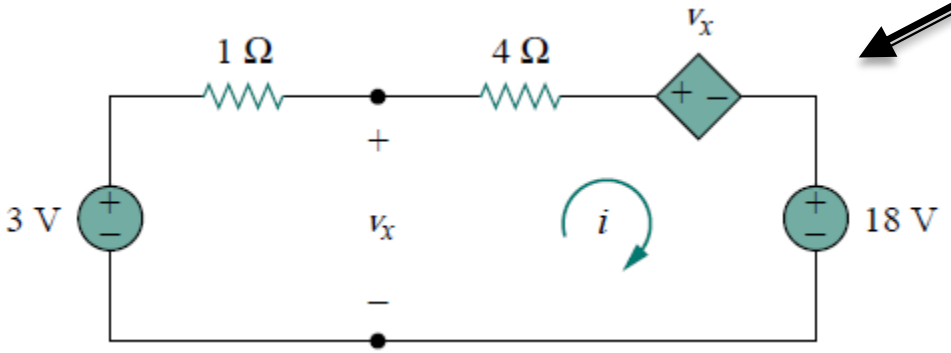
## ❑ Source Transformations

**Example:** Find $v_x$ using source transformation?



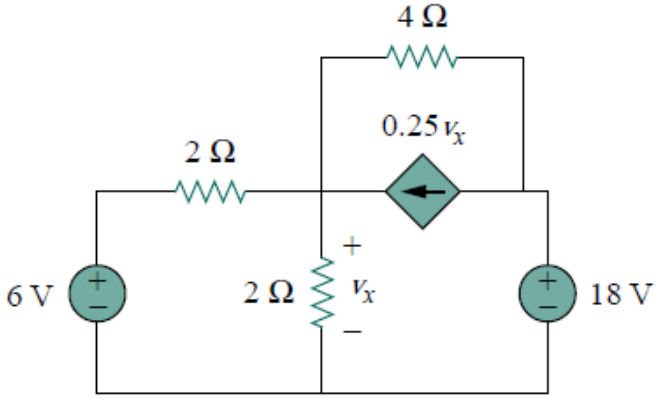



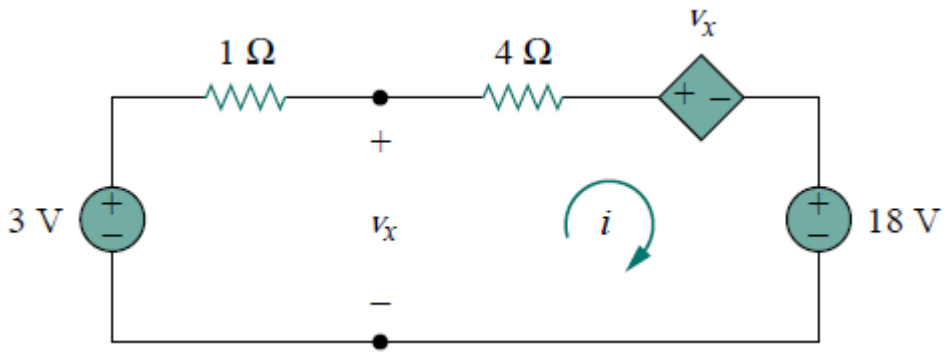




$$-3 + 5i + v_x + 18 = 0$$

## ❑ Source Transformations

**Example:** Find $v_x$ using source transformation?




$$-3 + 5i + v_x + 18 = 0$$

Applying KVL to the loop containing only the 3-V voltage source, the 1-Ω resistor, and $v_x$ yields

$$-3 + 1i + v_x = 0 \quad \Longrightarrow \quad v_x = 3 - i$$

Substituting this into Eq. , we obtain $15 + 5i + 3 - i = 0 \quad \Longrightarrow \quad i = -4.5$ A

Thus, $v_x = 3 - i = 7.5$ V.

## ❑ Superposition Principle

❑ The idea of superposition rests on the **linearity** property.

❑ The superposition principle states that the voltage across (or current through) an element in a **linear** circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

❑ The principle of superposition helps us to analyze a **linear** circuit with more than one independent source by calculating the contribution of each independent source separately.

## ❑ Superposition Principle

❑ To apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are ***turned off***.

   *Voltage sources are short-circuited (**V=0**)*

   *Current sources are open-circuited (**i=0**)*

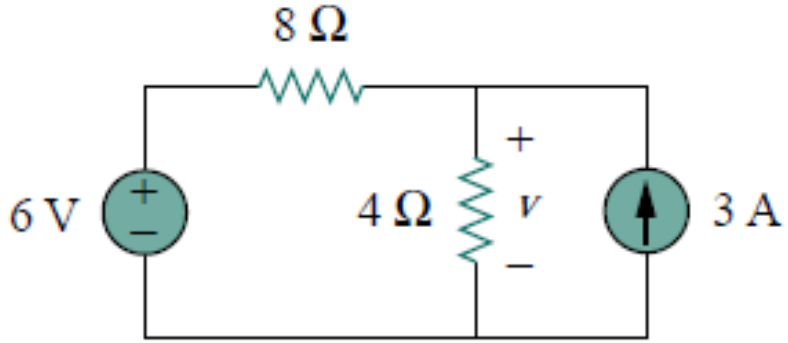
2. Dependent sources are left intact because they are controlled by circuit variables.

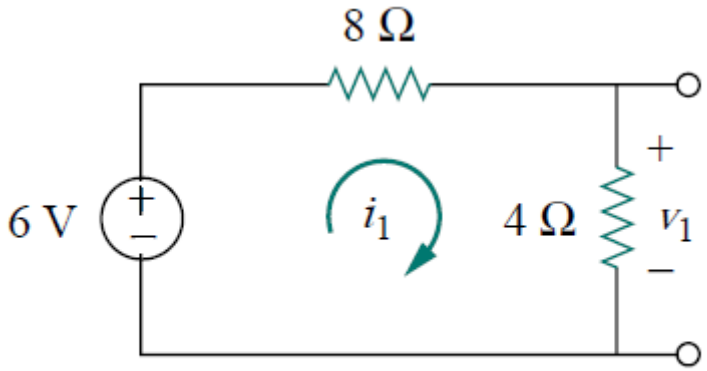
## ❑ Steps to apply the superposition principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

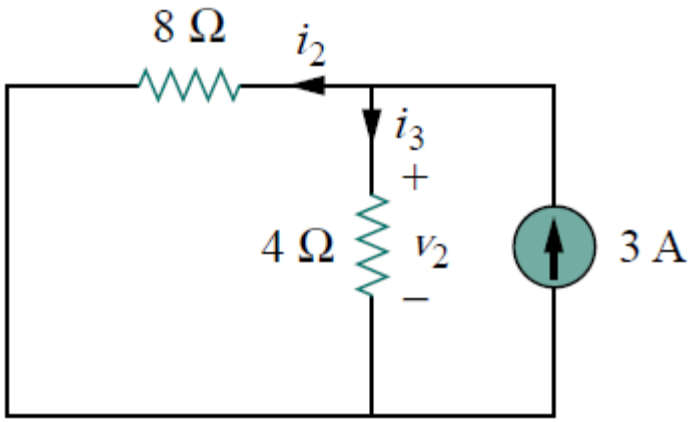
## ❑ Superposition Principle

**Example:** Use the superposition theorem to find $v$ *in the circuit below.*



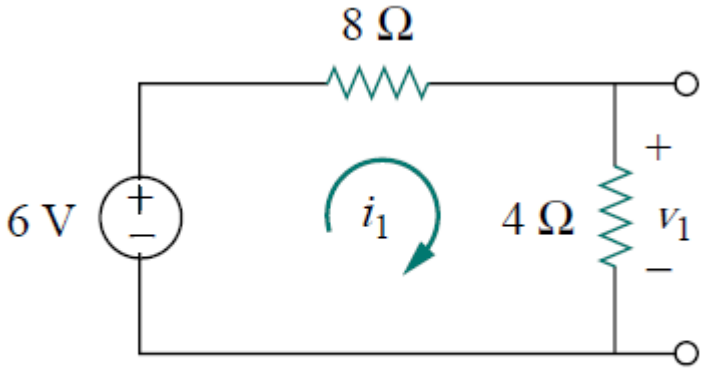

(a)



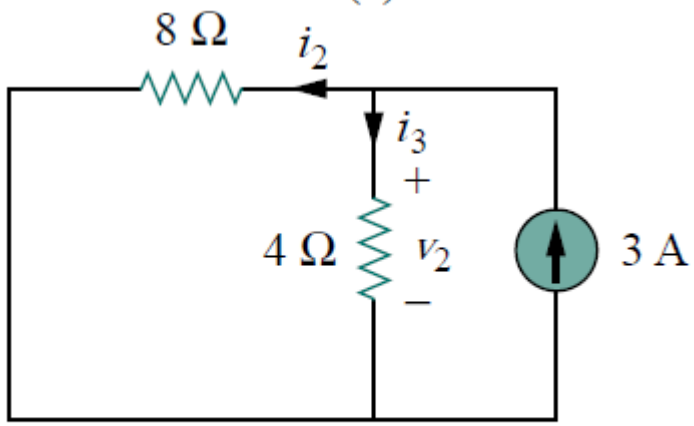

(b)

## ❑ Superposition Principle

**Example:** Use the superposition theorem to find $v$ *in the circuit below.*




(a)




(b)

$$12i_1 - 6 = 0 \quad \Longrightarrow \quad i_1 = 0.5 \text{ A}$$

$$v_1 = 4i_1 = 2 \text{ V}$$

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

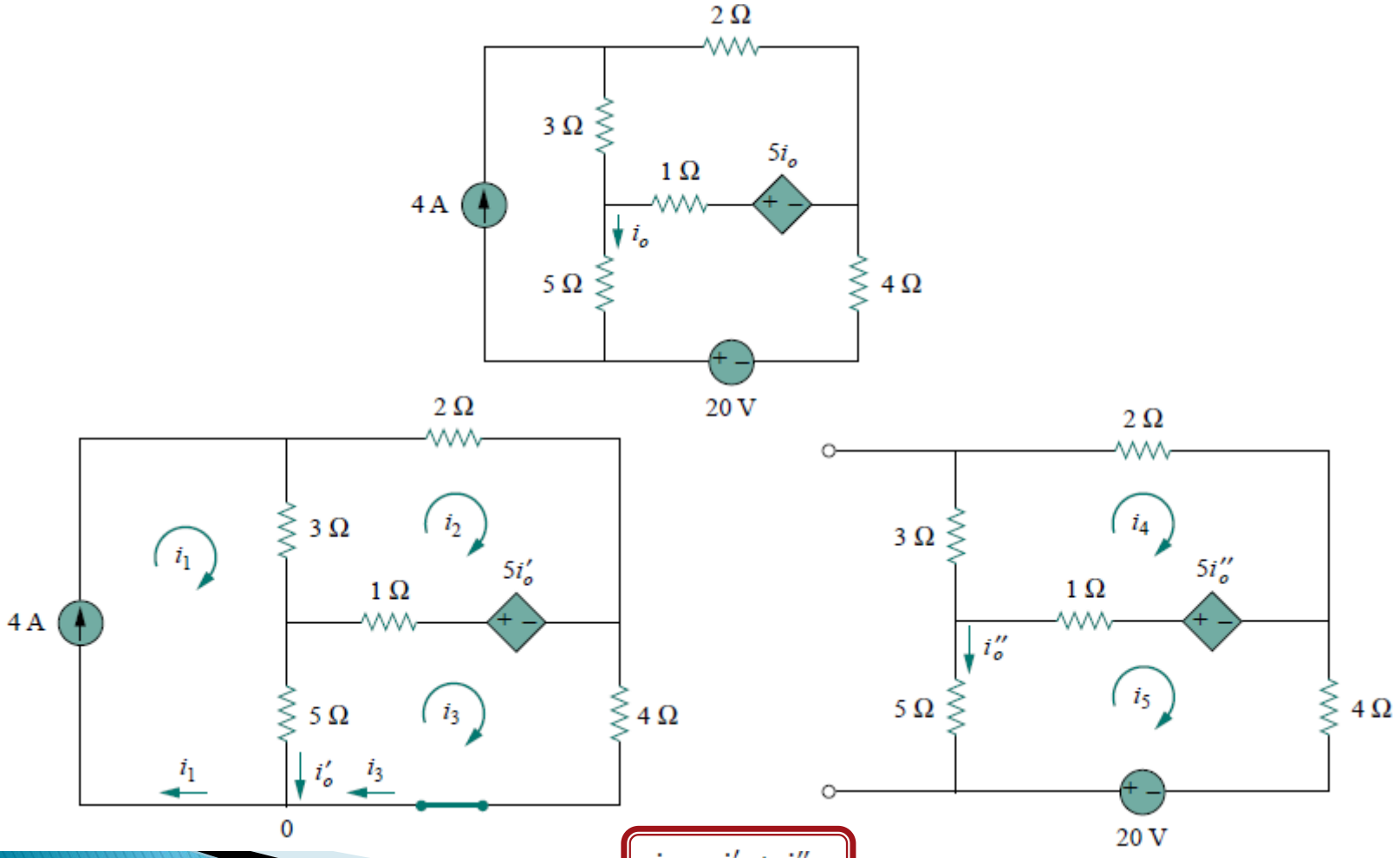
$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

$$v_2 = 4i_3 = 8 \text{ V}$$

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

## ❑ Superposition Principle

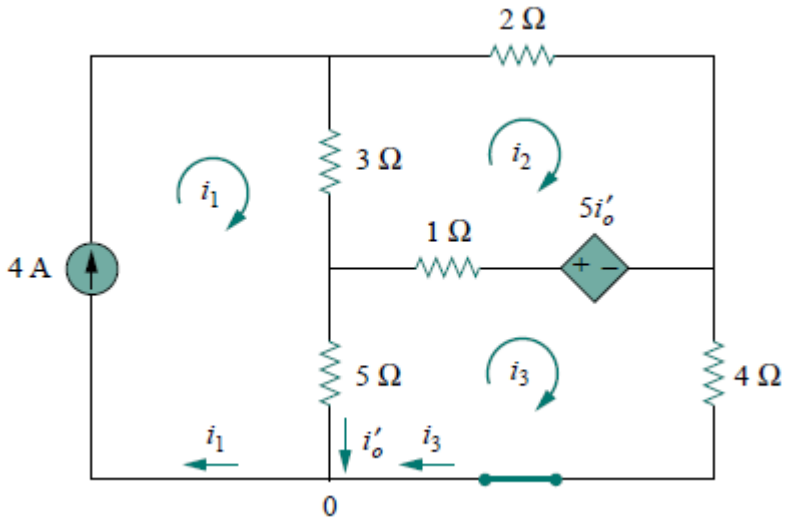
**Example:** Use the superposition theorem to find $i_0$ *in the circuit below.*




$$i_o = i_o' + i_o''$$

## ❑ Superposition Principle

**Example:** Use the superposition theorem to find $i_0$ *in the circuit below.*




$i_1 = 4$ A

For loop 2, $-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0$

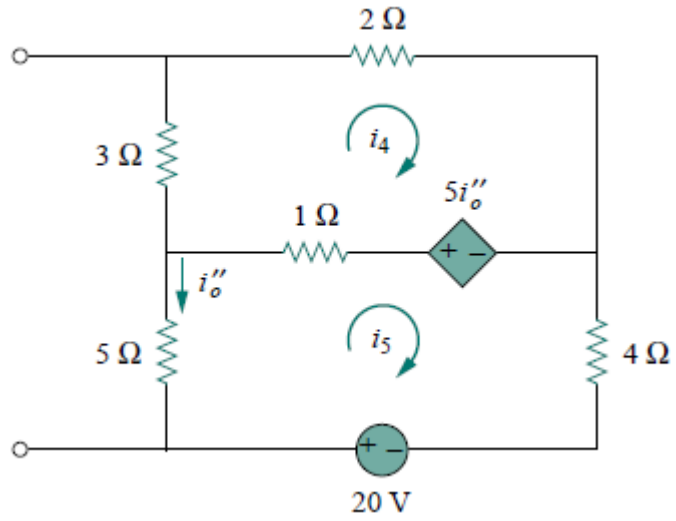
For loop 3, $-5i_1 - 1i_2 + 10i_3 + 5i'_o = 0$

But at node 0, $i_3 = i_1 - i'_o = 4 - i'_o$

$$i'_o = \frac{52}{17}\ \text{A}$$

## ❑ Superposition Principle

**Example:** Use the superposition theorem to find $i_0$ *in the circuit below.*

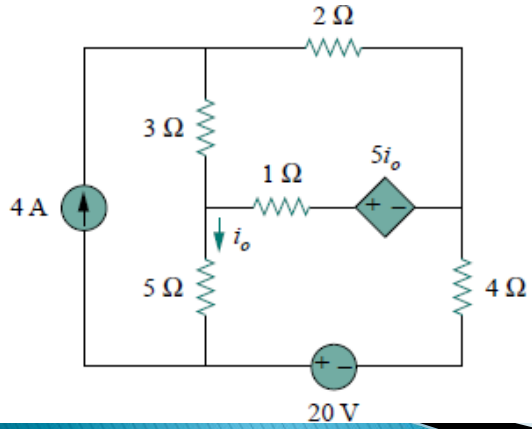



For loop 4, KVL gives $6i_4 - i_5 - 5i_o'' = 0$

for loop 5, $-i_4 + 10i_5 - 20 + 5i_o'' = 0$

But $i_5 = -i_o''$.

$$i_o'' = -\frac{60}{17}\ \text{A}$$



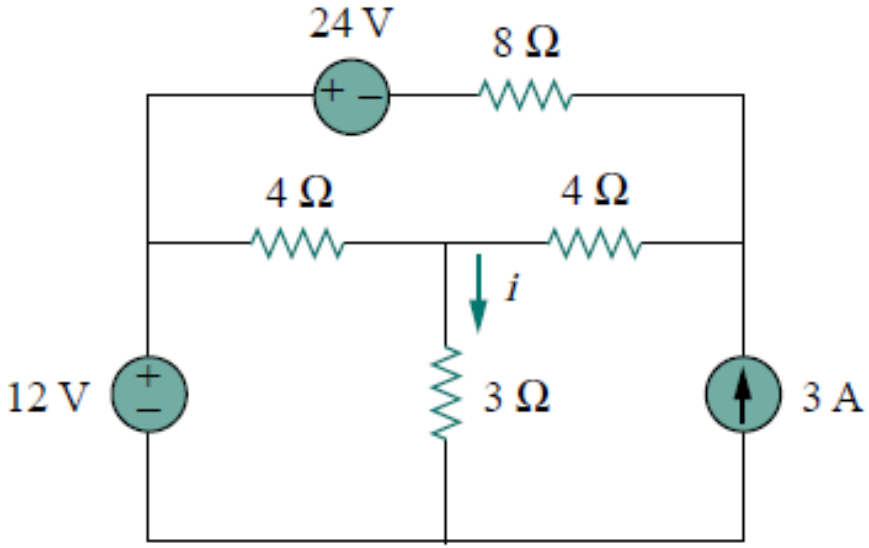

$$i_o' = \frac{52}{17}\ \text{A}$$

$$i_o'' = -\frac{60}{17}\ \text{A}$$

$$\Rightarrow\quad i_o = -\frac{8}{17} = -0.4706\ \text{A}$$

## ❑ Superposition Principle

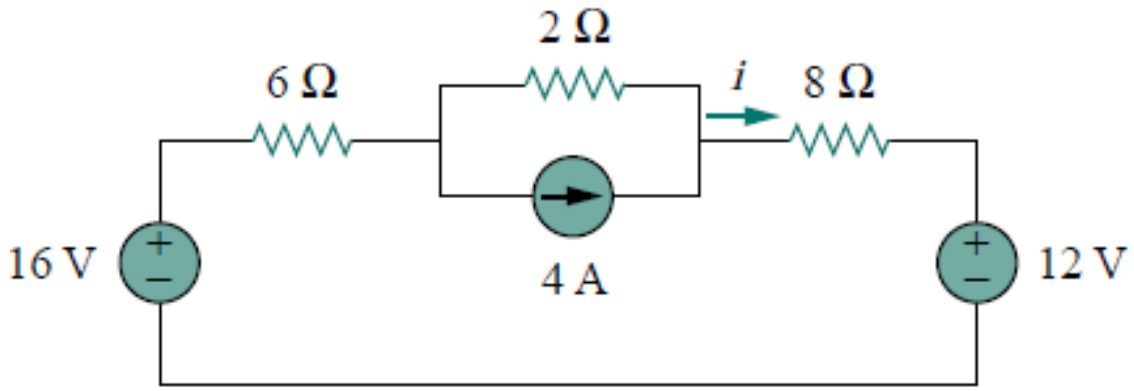
**Practice Problem:** Use the superposition theorem to find $i$ *in the circuit below.*




$$i = 2A$$

## ❑ Superposition Principle

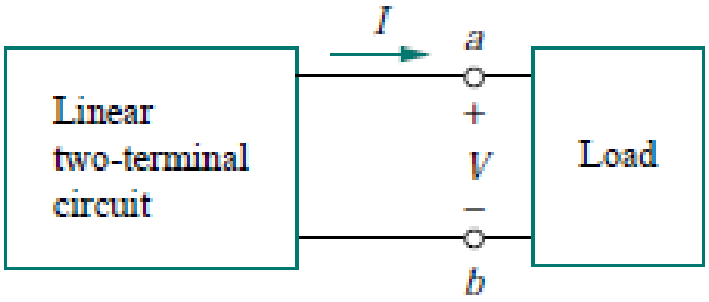
**Practice Problem:** Use the superposition theorem to find $i$ *in the circuit below.*




$$i = 0.75A$$

## ❑ Thevenin's & Norton's Theorems

❑ **Linearity** is the property of an element describing a linear relationship between cause and effect. The property is a combination of both the **homogeneity** (scaling) property and the **additivity** property.




homogeneity

$$v_i \Rightarrow v_0$$
$$k.v_i \Rightarrow kv_0$$

$$v_a \Rightarrow v_1, v_b \Rightarrow v_2$$
$$v_a + v_b \Rightarrow v_1 + v_2$$

additivity

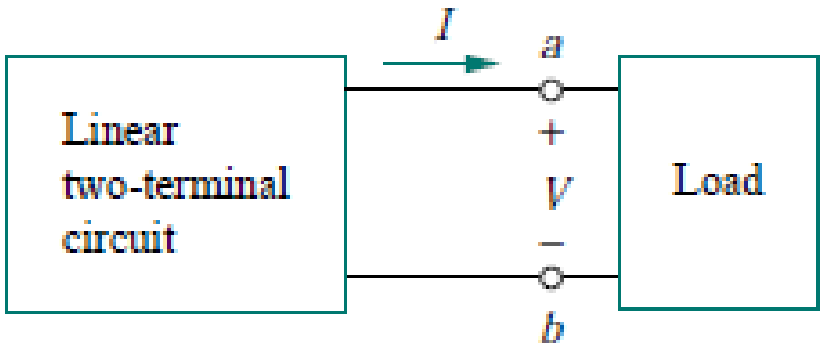
**In general, a linear system must hold**

$$v_a \Rightarrow v_1, v_b \Rightarrow v_2$$
$$k_1 v_a + k_2 v_b \Rightarrow k_1 v_1 + k_2 v_2$$

❑ **A linear circuit is one whose output is linearly related (or directly proportional) to its input.**

## ❑ Thevenin's & Norton's Theorems

❑ It often occurs in practice that a particular element in a circuit is variable (usually called the *load) while other elements are fixed.*



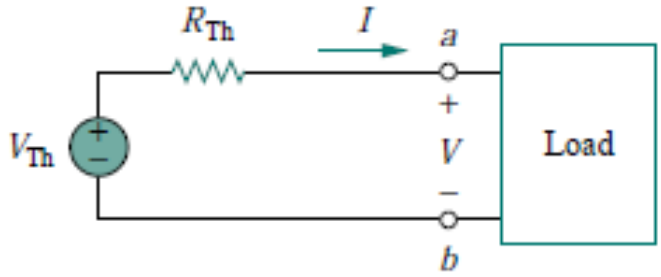

❑ At times in circuit analysis, we want to concentrate on what happens at a specific pair of terminals.

## ❑ Thevenin's & Norton's Theorems

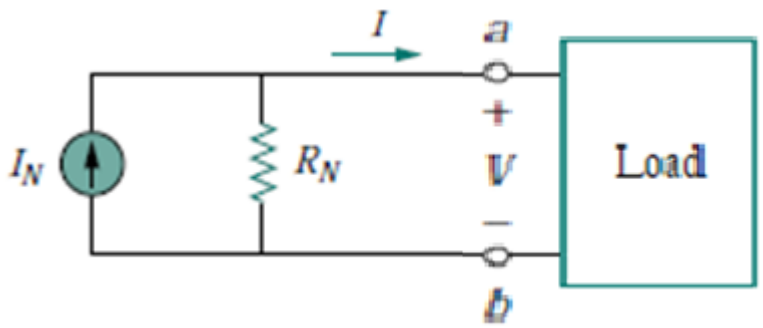
❑ *As a typical* example, a household outlet terminal may be connected to different appliances constituting a variable load.

❑ Each time the variable element is changed, the entire circuit has to be analyzed all over again.

❑ **To avoid this problem, Thevenin's and Norton's theorems provide a technique by which the fixed part of the circuit is replaced by an equivalent circuit.**




**Thevenin equivalent**




**Norton equivalent**

## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

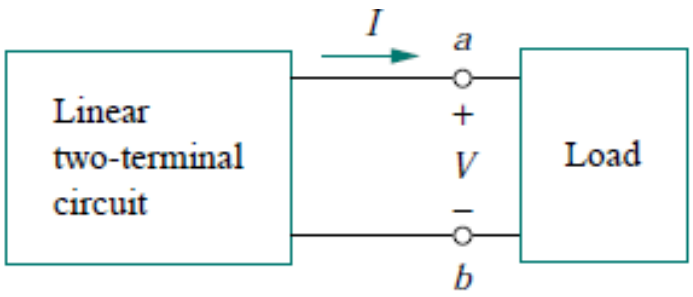


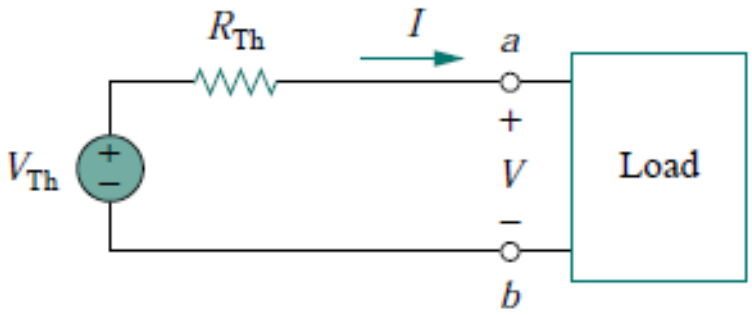
❑ Developed @ 1883 by French telegraph engineer Leon Charles Thevenin.

❑ Thevenin's theorem states that a linear two-terminal circuit can be **replaced by an equivalent circuit** consisting of a voltage source $V_{Th}$ *in series with* a resistor $R_{Th}$, *where* $V_{Th}$ *is the open-circuit voltage at the terminals* and $R_{Th}$ *is the input or equivalent resistance at the terminals when* the independent sources are turned off.

❑ Our major goal is to find the Thevenin equivalent circuit. $V_{Th}$ & $R_{Th}$

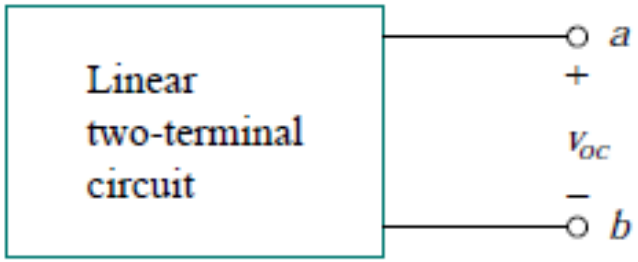
## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)




❑ To find the Thevenin equivalent voltage $(V_{Th})$

1. **Make open-circuit to terminals a-b (by removing the load)**
2. **Find the voltage across the open-circuited terminals a-b**




$$V_{Th} = v_{oc}$$

## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

❑ To find the Thevenin equivalent resistance ($R_{Th}$)

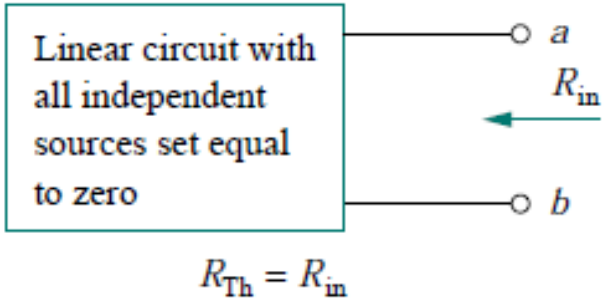
***If circuit does not include dependent sources (we have 2 methods)***

**1.i . Make open-circuit to terminals a-b**

**1.ii . Kill (turn off) all independent sources (v=0, i=0)**

**1.iii. Then,**

$$R_{Th} = R_{ab}$$

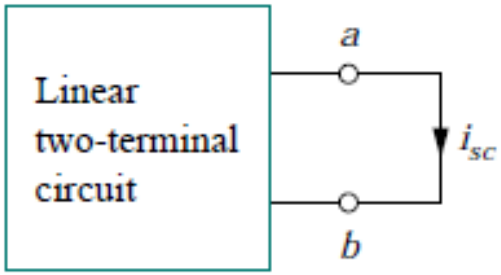



**2.i. Make short-circuit to terminals a-b**

**2.ii. Find the short-circuit current $i_{sc}$**

**2.iii. Then,**

$$R_{Th} = V_{Th} / i_{sc}$$

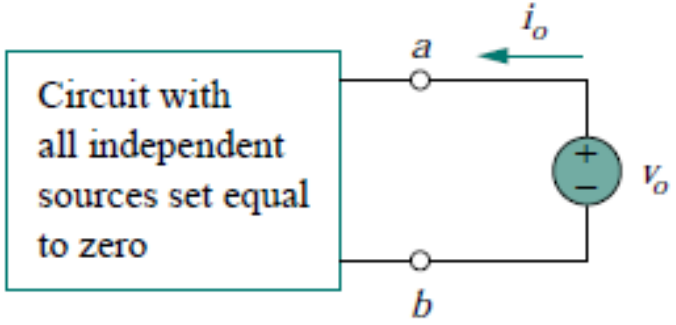


## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

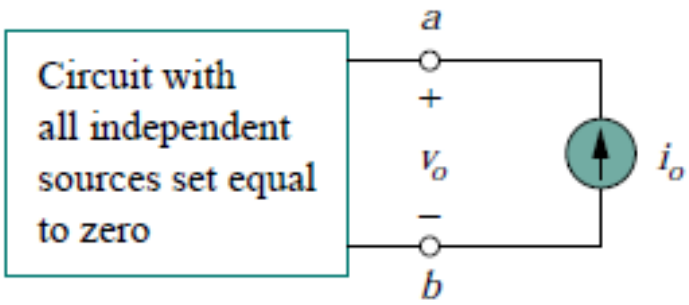
❑ To find the Thevenin equivalent resistance $(R_{Th})$

***If circuit include dependent sources***

1. **Make open-circuit to terminals a-b**
2. **Kill (turn off) all independent sources (v=0, i=0)**
3. **Apply a test voltage source $v_0$ (or current source $i_0$) between the terminals a-b**
4. **Find the unknown test current $i_0$ (or test voltage $v_0$ )**
5. **Then,**

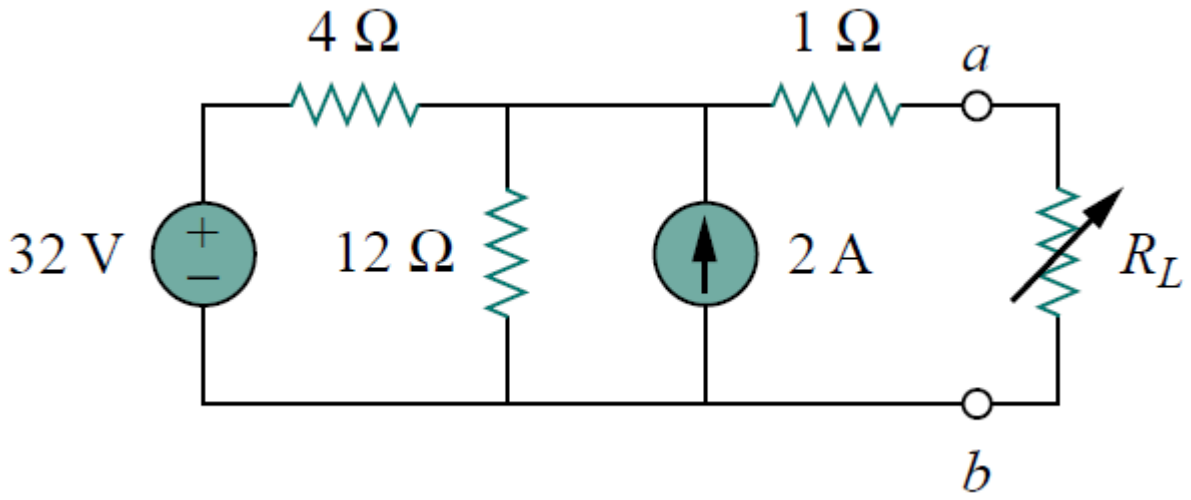



$$R_{Th} = \frac{v_0}{i_0}$$



## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

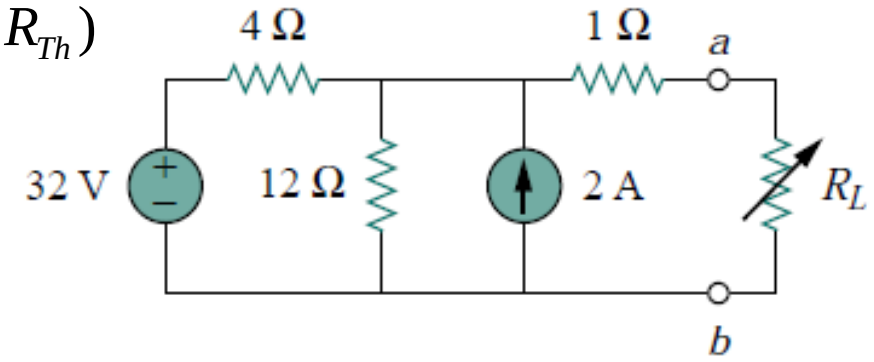
**Example:** Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals a-b. Then, find the current through load resistor when $R_L = 6\Omega, 16\Omega$ and $36\Omega$



## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

**Solution:** Find the Thevenin equivalent circuit when $R_L = 6\Omega, 16\Omega$ and $36\Omega$

❑ To find the Thevenin equivalent resistance $(R_{Th})$

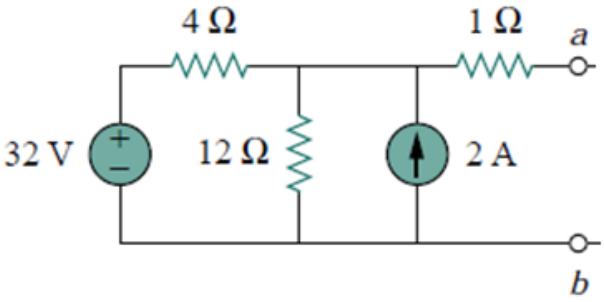


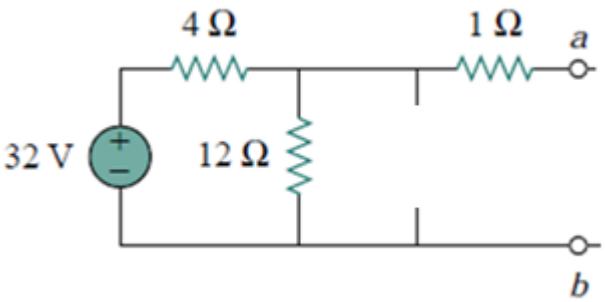

***If circuit does not include dependent sources***

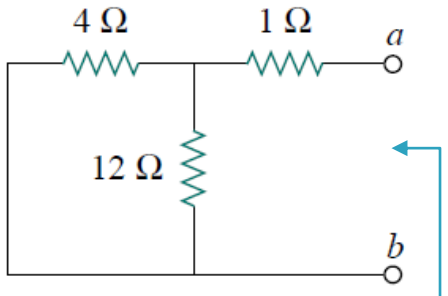
**1.i . Make open-circuit to terminals a-b**
**(by removing the load)**

**1.ii . Kill (turn off) all independent sources (v=0, i=0)**

**1.iii. Then,** $R_{Th} = R_{ab}$




$R_{ab} = R_{Th}$

$$R_{\text{Th}} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

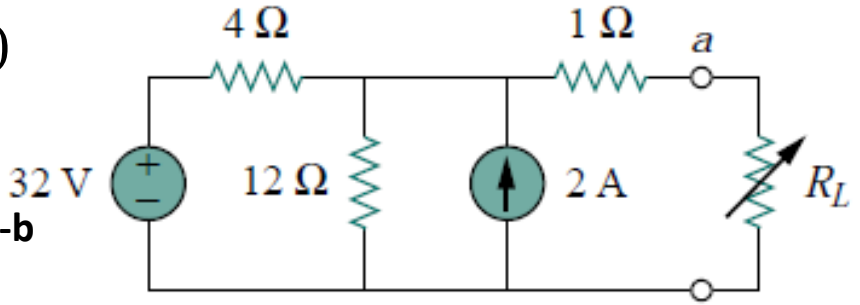
## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

**Solution:** Find the Thevenin equivalent circuit when $R_L = 6\Omega, 16\Omega$ and $36\Omega$

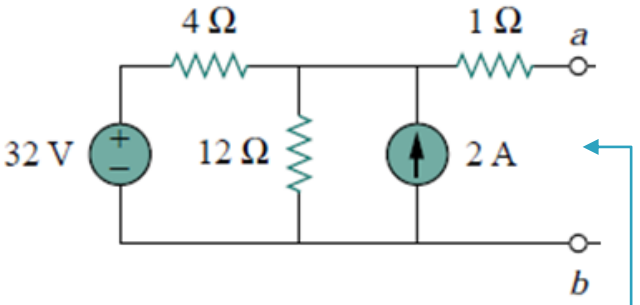
❑ To find the Thevenin equivalent voltage ($V_{Th}$)

**1. Make open-circuit to terminals a-b**
**(by removing the load)**

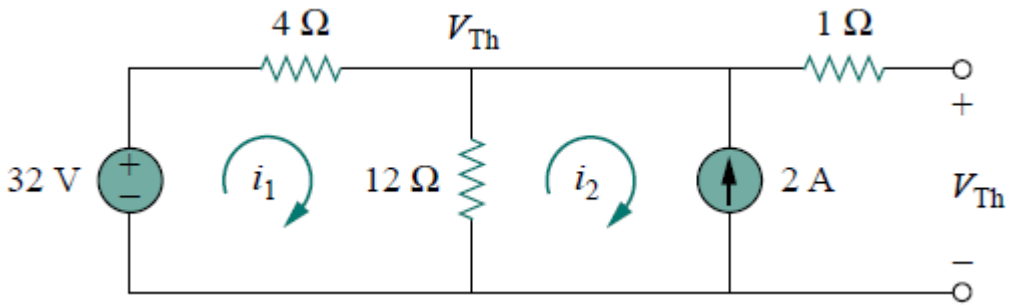
**2. Find the voltage across the open-circuited terminals a-b**




$$V_{ab} = V_{Th}$$




$$V_{ab} = V_{Th}$$




Applying mesh analysis to the two loops, we obtain

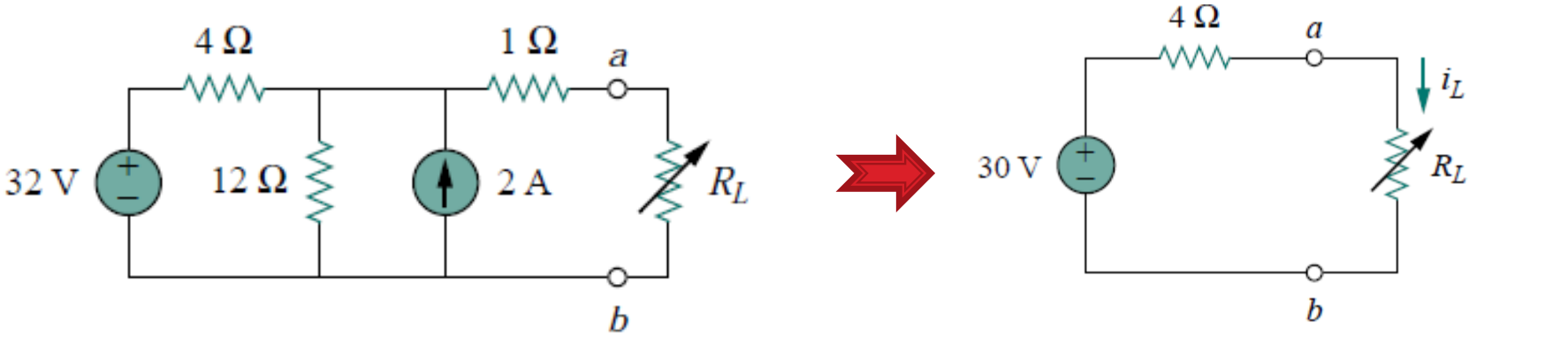
$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2 \text{ A}$$

Solving for $i_1$, we get $i_1 = 0.5$ A. Thus,

$$V_{\text{Th}} = 12(i_1 - i_2)$$
$$= 12(0.5 + 2.0)$$
$$= 30 \text{ V}$$

## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

**Solution:** Find the current through load resistor when $R_L = 6\Omega, 16\Omega$ and $36\Omega$




The Thevenin equivalent circuit

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{30}{4 + R_L}$$

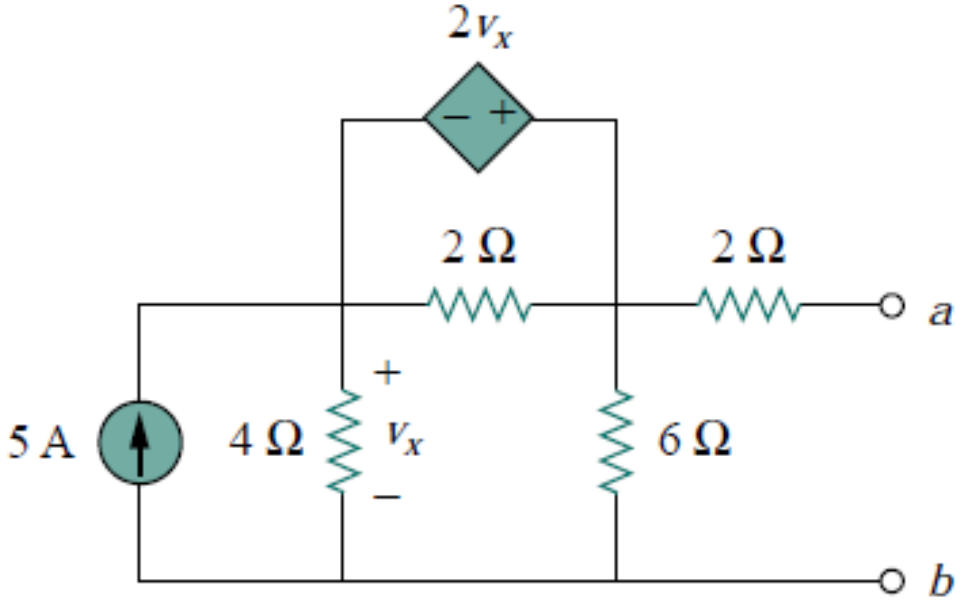
When $R_L = 6$, $\Rightarrow$ $I_L = \dfrac{30}{10} = 3$ A

When $R_L = 16$, $\Rightarrow$ $I_L = \dfrac{30}{20} = 1.5$ A

When $R_L = 36$, $\Rightarrow$ $I_L = \dfrac{30}{40} = 0.75$ A

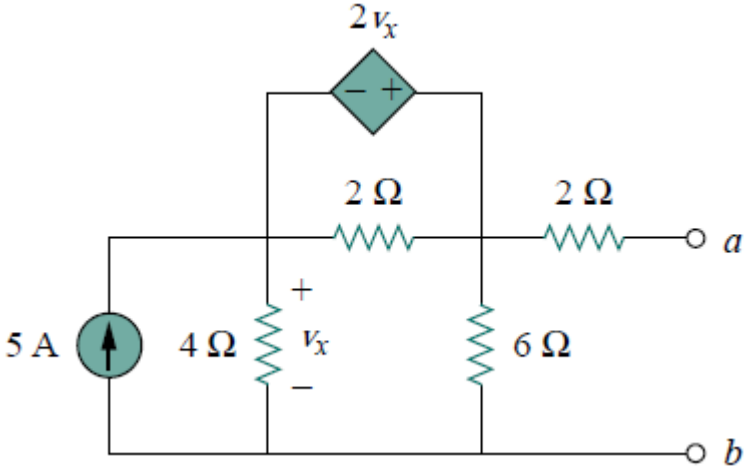
## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

**Example:** Find the Thevenin equivalent circuit of the circuit shown?

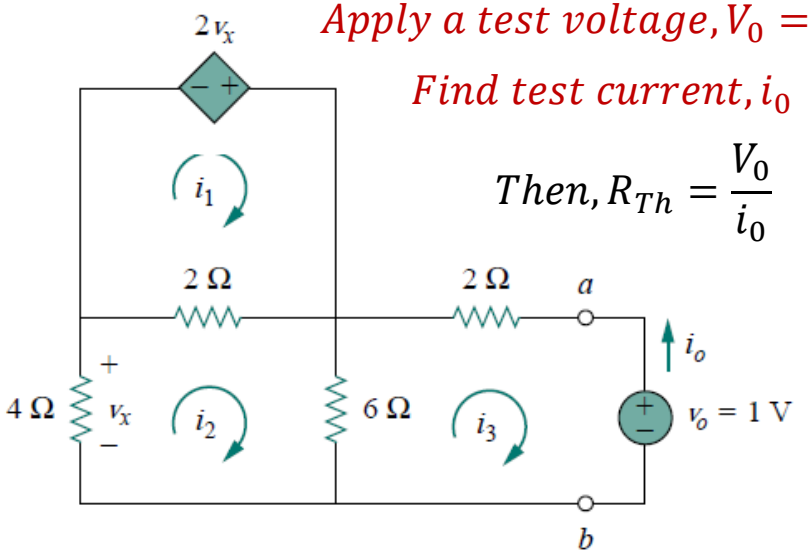


## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

**Example:** Find the Thevenin equivalent circuit of the circuit shown?




*for* $R_{Th}$

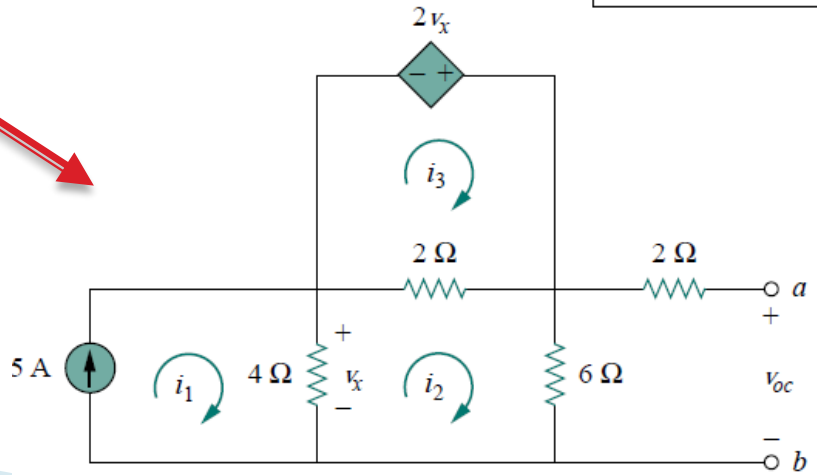



*Apply a test voltage,* $V_0 = 1V$

*Find test current,* $i_0$

$$Then, R_{Th} = \frac{V_0}{i_0}$$

*for* $V_{Th}$

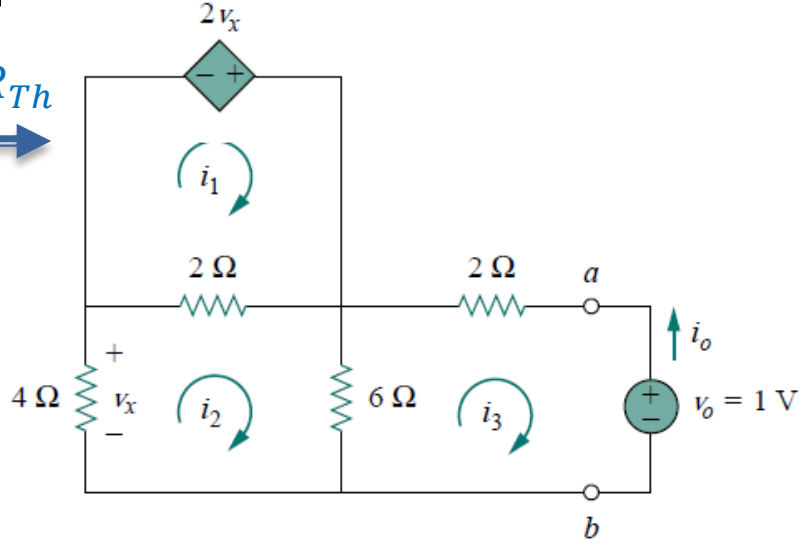



$$V_{Th} = V_{oc}$$

## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

**Example:** Find the Thevenin equivalent circuit of the circuit shown?

$for\ R_{Th}$




Applying mesh analysis to loop 1

$$-2v_x + 2(i_1 - i_2) = 0 \quad \text{or} \quad v_x = i_1 - i_2$$

But $-4i_2 = v_x = i_1 - i_2$; hence,

$$i_1 = -3i_2$$

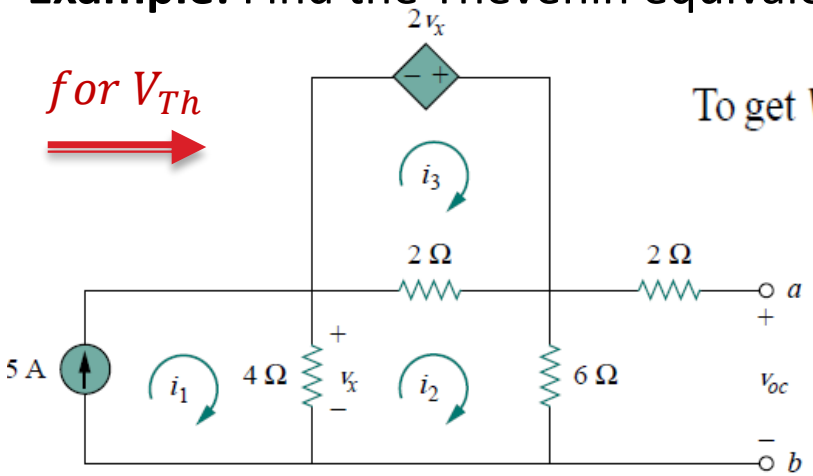
For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$
$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

Solving these equations gives $i_3 = -\frac{1}{6}$ A But $i_o = -i_3 = 1/6$ A. Hence, $R_{\text{Th}} = \frac{1\text{ V}}{i_o} = 6\ \Omega$

## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

**Example:** Find the Thevenin equivalent circuit of the circuit shown?




*for* $V_{Th}$

To get $V_{\text{Th}}$, we find $v_{oc}$ in the circuit. Applying mesh analysis, we get

$$i_1 = 5$$

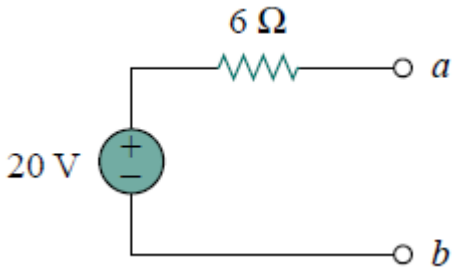
$$-2v_x + 2(i_3 - i_2) = 0 \quad \Longrightarrow \quad v_x = i_3 - i_2$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

$$\text{or} \quad 12i_2 - 4i_1 - 2i_3 = 0$$

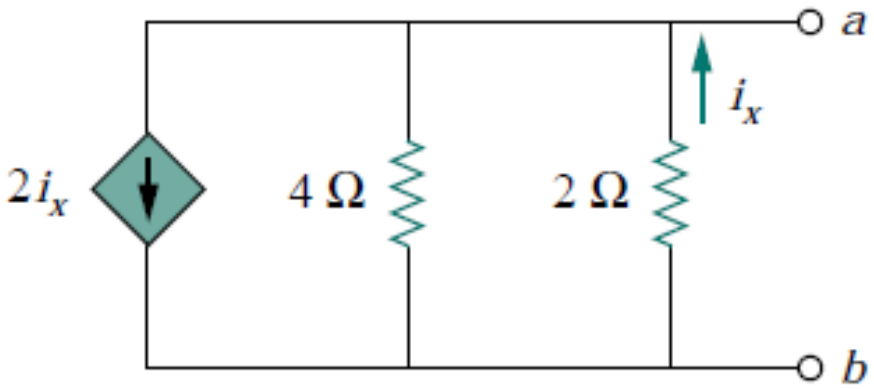
But $4(i_1 - i_2) = v_x$. Solving these equations leads to $i_2 = 10/3$. Hence,

$$V_{\text{Th}} = v_{oc} = 6i_2 = 20 \text{ V}$$



## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

**Example:** Find the Thevenin equivalent circuit of the circuit shown?

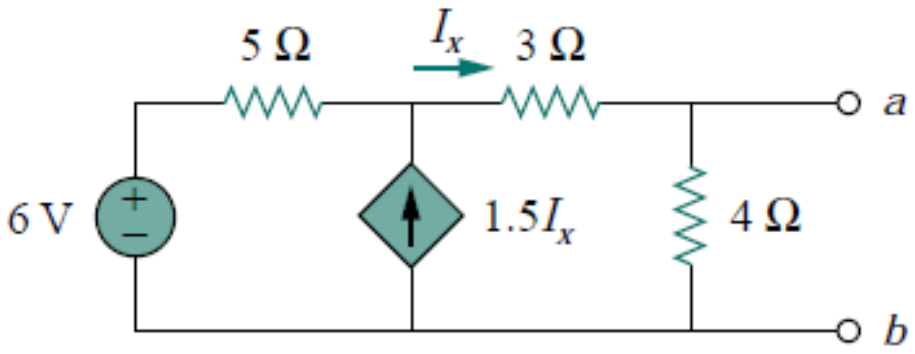



$$V_{Th} = 0V$$

$$R_{Th} = -4\Omega$$

## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

**Practice Problem:** Find the Thevenin equivalent circuit of the circuit shown?

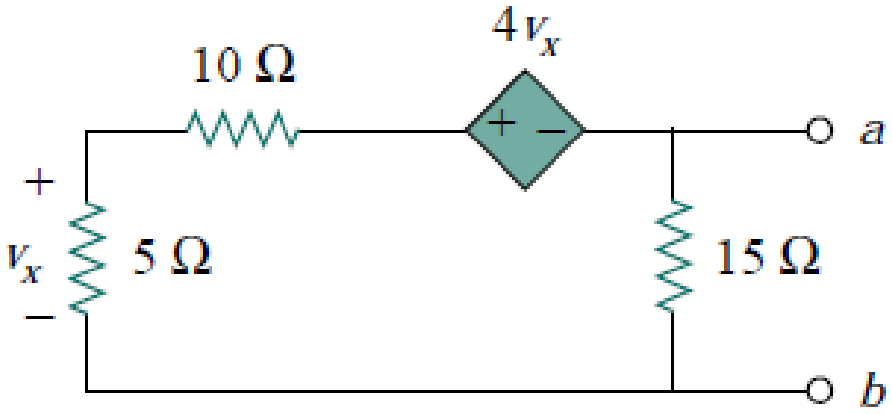



$$V_{Th} = 5.33V$$

$$R_{Th} = 0.44\Omega$$

## ❑ Thevenin's Theorem (Thevenin equivalent Circuit)

**Practice Problem:** Find the Thevenin equivalent circuit of the circuit shown?




$$V_{Th} = 0V$$

$$R_{Th} = -7.5\Omega$$

## ❑ Norton's Theorem (Norton equivalent Circuit)



❑ In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

❑ Norton's theorem states that a linear two-terminal circuit can be **replaced by an equivalent circuit** consisting of a current source $I_N$ *in parallel with* a resistor $R_N$ , *where* $I_N$ *is the short-circuit current through the terminals* and $R_N$ *is the input or equivalent resistance at the terminals when* the independent sources are turned off.

❑ Our major goal is to find the Norton equivalent circuit. $I_N$ & $R_N$

## ❑ Norton's Theorem (Norton equivalent Circuit)

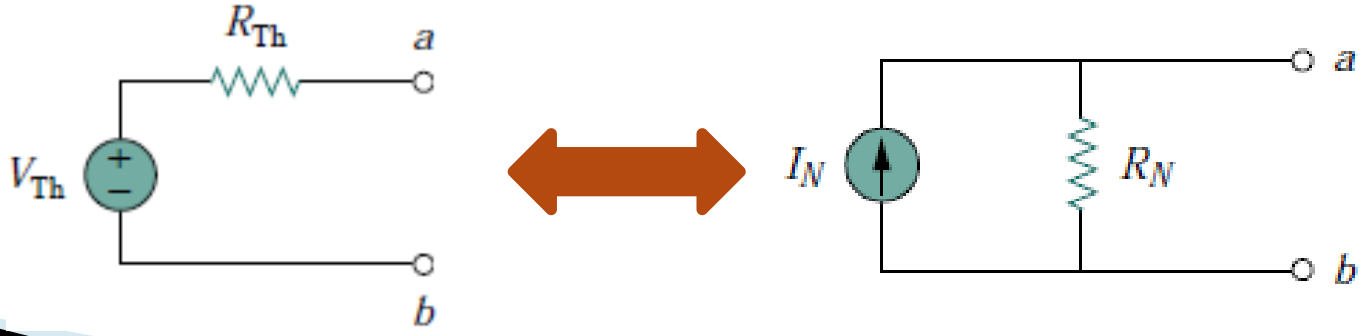
❑ Finding the Norton equivalent circuit is simple (As you learned to obtain Thevenin equivalent circuit)

$$R_N = R_{Th}$$

$$I_N = i_{sc}$$

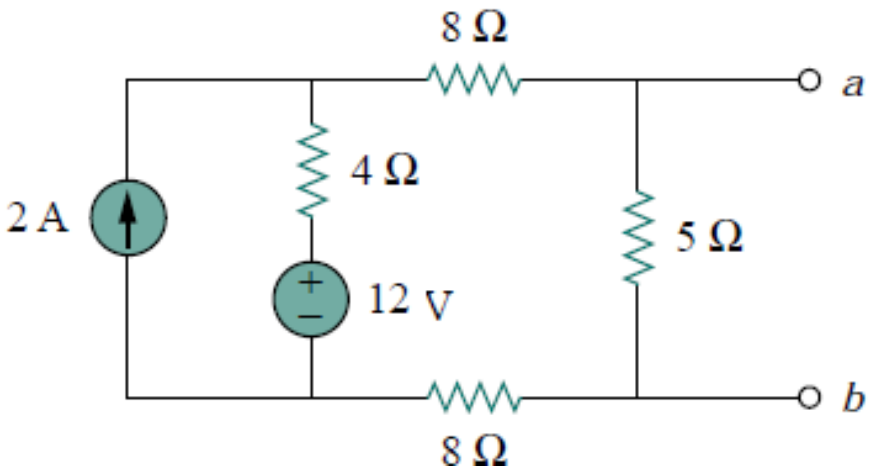
*As we found in Thevenin method to calculate $R_{Th}$

❑ One can **use source transformation** to obtain Norton equivalent circuit from Thevenin equivalent one.



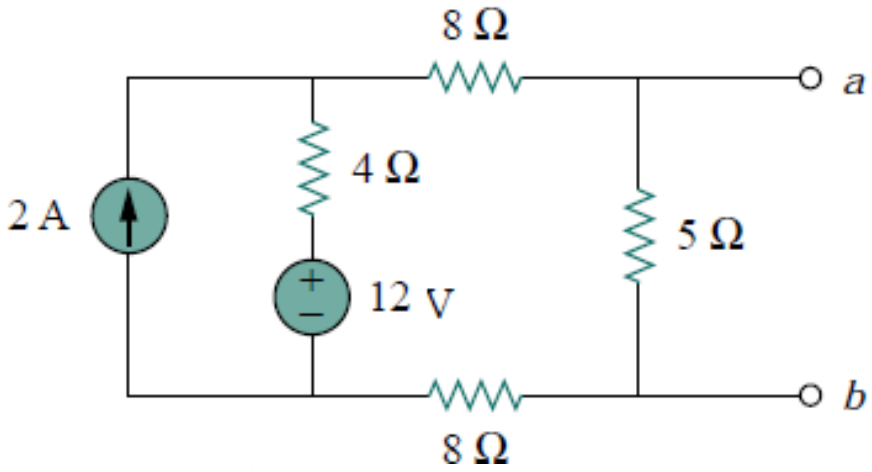
## ❑ Norton's Theorem (Norton equivalent Circuit)

**Example:** Find the Norton equivalent circuit of the circuit shown?

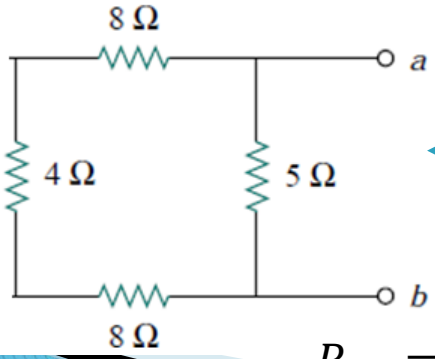


## ❑ Norton's Theorem (Norton equivalent Circuit)

**Example:** Find the Norton equivalent circuit of the circuit shown?

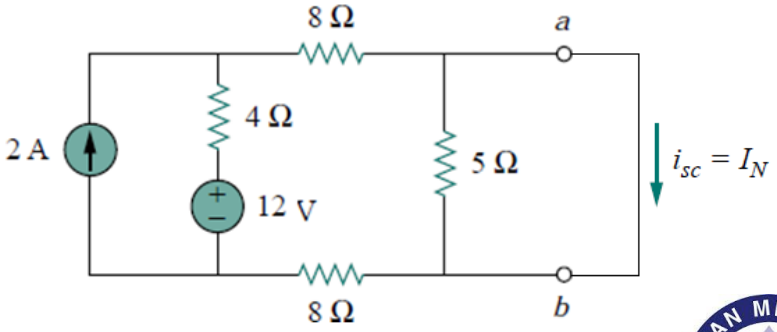



❑ To find the Norton equivalent resistance $R_N$ $(R_N = R_{Th})$



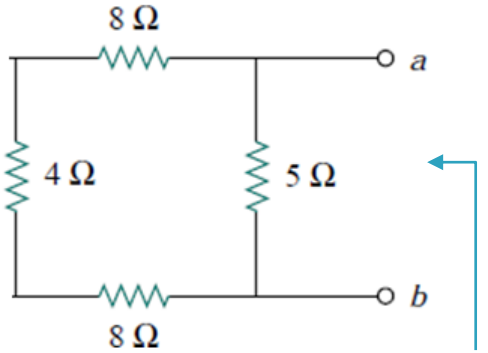

$$R_{ab} = R_N = R_{Th}$$

❑ To find the Norton equivalent current $I_N$ $(I_N = I_{sc})$



## ❑ Norton's Theorem (Norton equivalent Circuit)

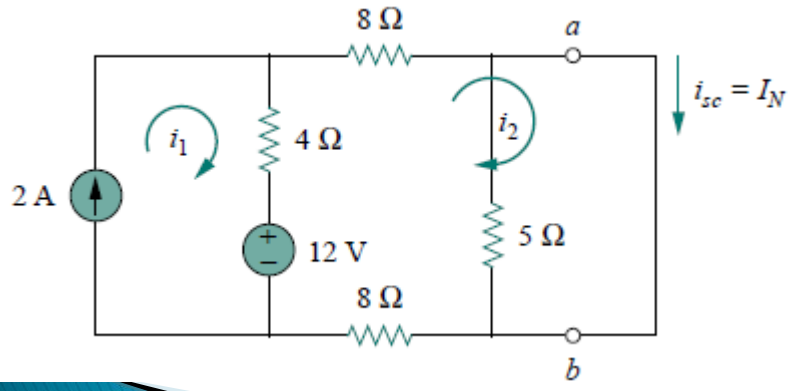
❑ To find the Norton equivalent resistance $R_N$ $(R_N = R_{Th})$




$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

$$R_{ab} = R_N = R_{Th}$$

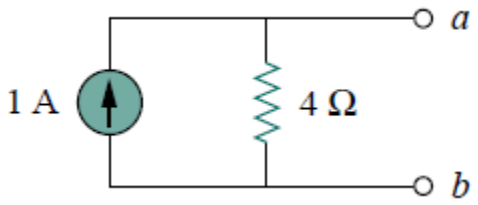
❑ To find the Norton equivalent current $I_N$ $(I_N = I_{sc})$




$$i_1 = 2\ \text{A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

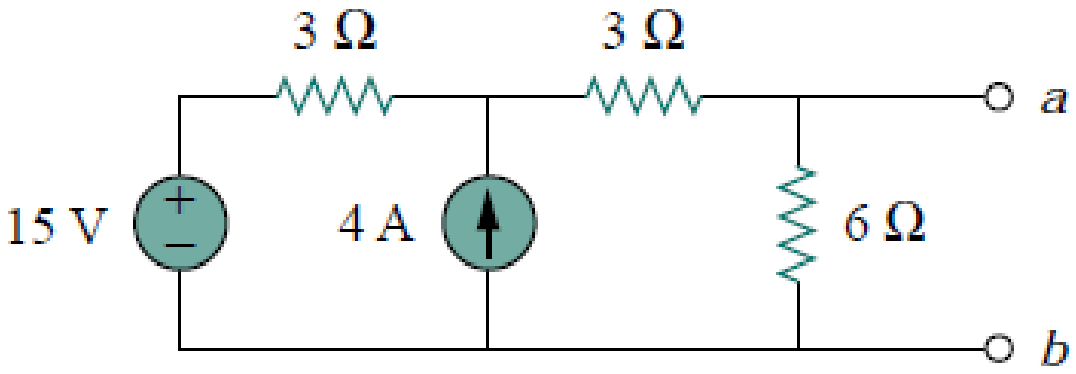
From these equations, we obtain

$$i_2 = 1\ \text{A} = i_{sc} = I_N$$



## ❑ Norton's Theorem (Norton equivalent Circuit)

**Practice Problem:** Find the Norton equivalent circuit of the circuit shown?

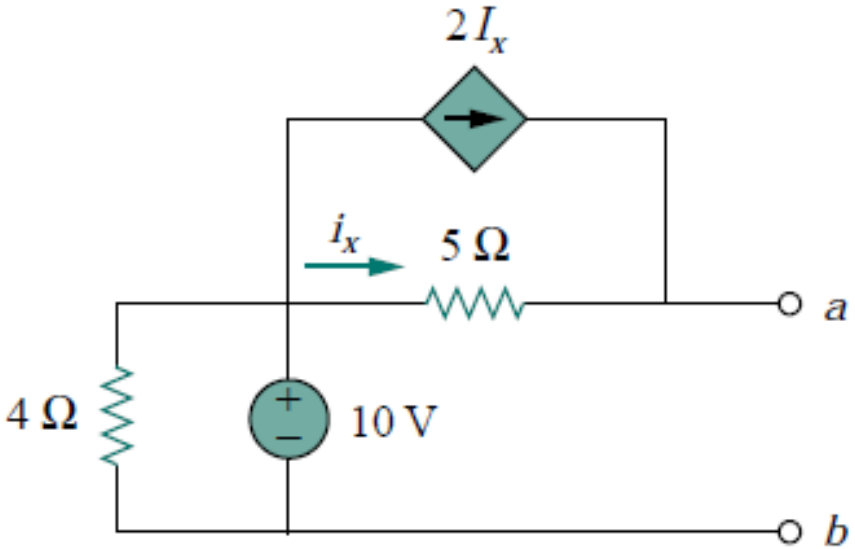



$$I_N = 4.5A$$

$$R_N = 3\Omega$$

## ❑ Norton's Theorem (Norton equivalent Circuit)

**Practice Problem:** Find the Norton equivalent circuit of the circuit shown?




$$R_N = -1.67\ \Omega$$

$$I_N = 6\ \text{A}$$

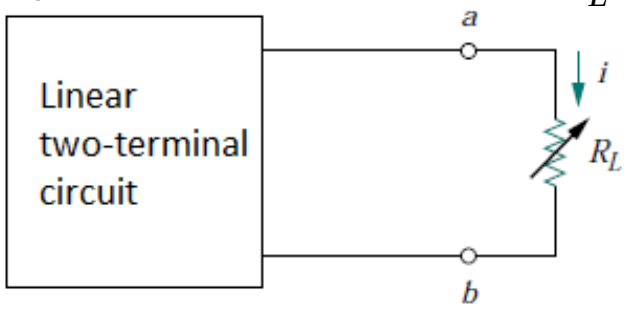
## ❑ Maximum Power Transfer

❑ In many practical situations, a circuit is designed to provide power to a load.

❑ While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load.

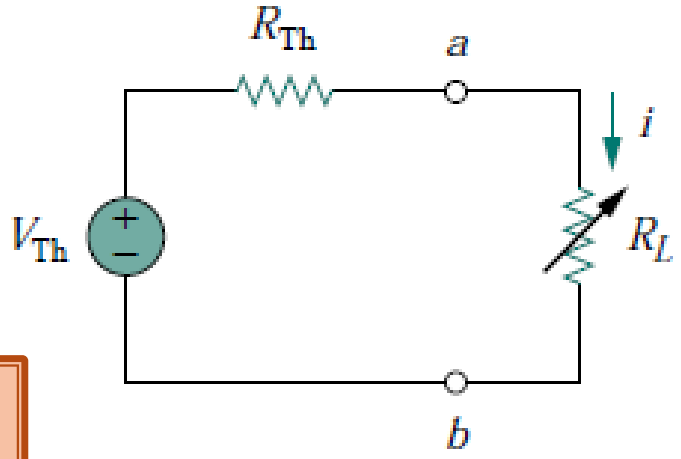
❑ The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load.

❑ We assume that we can adjust the load resistance $R_L$



## ❑ Maximum Power Transfer

❑ If the entire circuit is replaced by its Thevenin equivalent except for the load, the power delivered to the load is:




$$p = i^2 R_L = \left( \frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

❑ To find the value of load resistor when the power transferred to the resistor is maximum, we must differentiate the equation above w.r.t. $R_L$ .Then, resistance value can be found by equalizing these equations to zero.

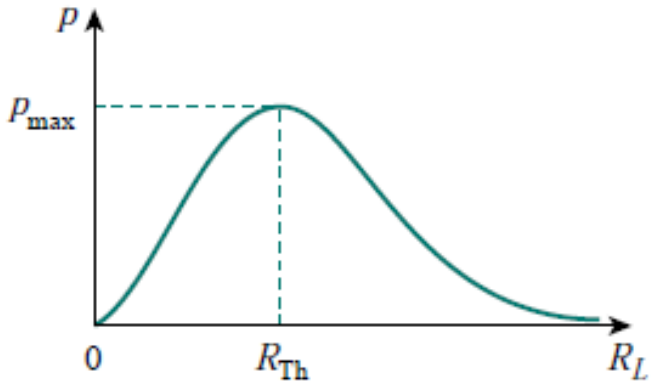
## ❑ Maximum Power Transfer

$$\frac{dp}{dR_L} = V_{\text{Th}}^2 \left[ \frac{(R_{\text{Th}} + R_L)^2 - 2R_L(R_{\text{Th}} + R_L)}{(R_{\text{Th}} + R_L)^4} \right]$$

$$= V_{\text{Th}}^2 \left[ \frac{(R_{\text{Th}} + R_L - 2R_L)}{(R_{\text{Th}} + R_L)^3} \right] = 0 \qquad 0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L)$$

$$R_L = R_{Th}$$

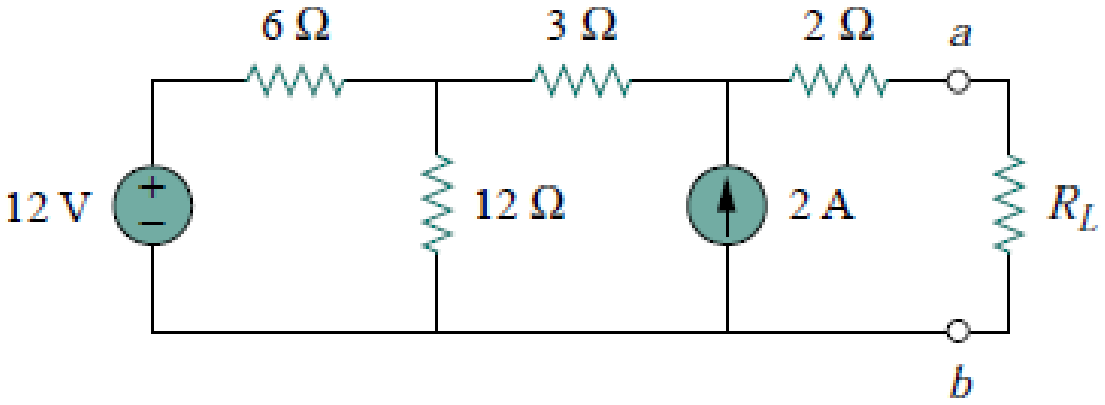
showing that the maximum power transfer takes place when the load resistance equals the Thevenin resistance. In this case, the source and load are said to be **matched**.




$$p_{\text{max}} = \frac{V_{Th}^2}{4R_{Th}}$$

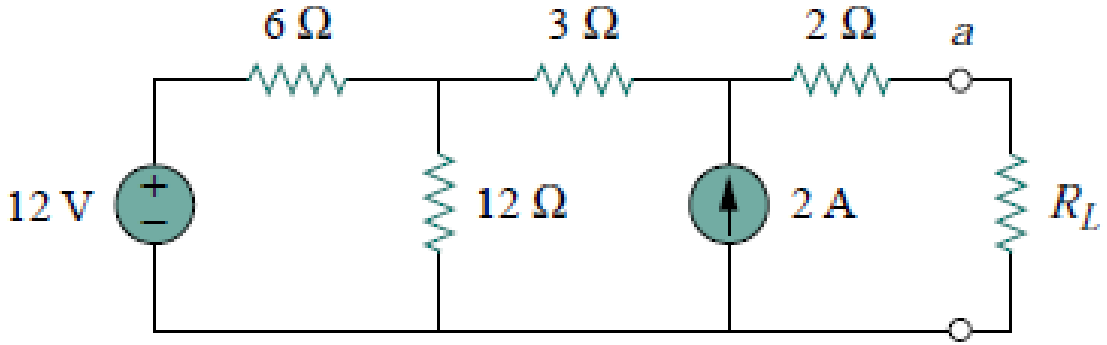
## ❑ Maximum Power Transfer

**Example:** Find the value of $R_L$ *for maximum power transfer in the circuit below*. Find the maximum power?



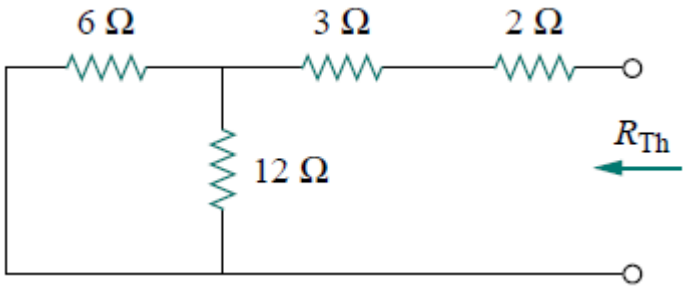
## ❑ Maximum Power Transfer

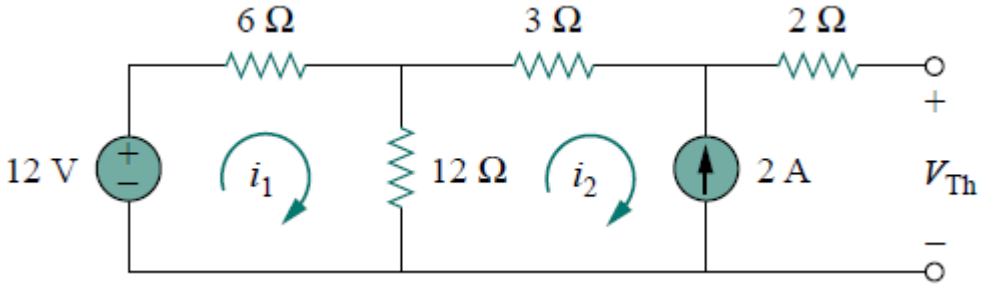
**Example:** Find the value of $R_L$ *for maximum power transfer in the circuit below*. Find the maximum power?




**Solution:**

We need to find the Thevenin resistance $R_{\text{Th}}$ and the Thevenin voltage $V_{\text{Th}}$ across the terminals $a$-$b$.

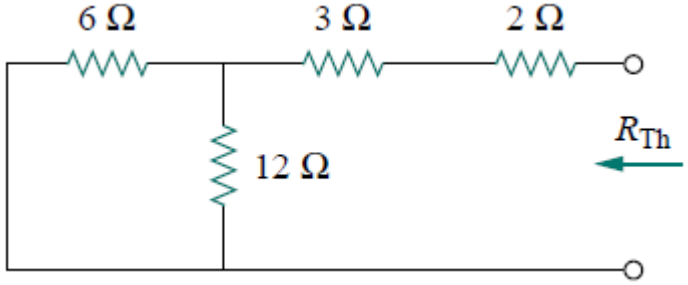


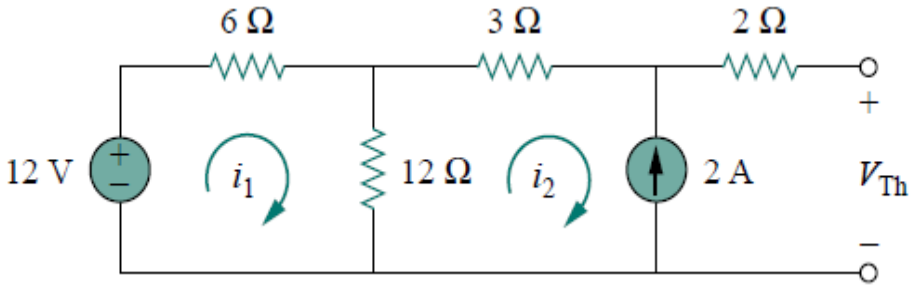
## ❑ Maximum Power Transfer

**Solution:**

We need to find the Thevenin resistance $R_{\text{Th}}$ and the Thevenin voltage $V_{\text{Th}}$ across the terminals $a$-$b$.




$$R_{\text{Th}} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$




Applying mesh analysis,

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\ \text{A}$$

Solving for $i_1$, we get $i_1 = -2/3$.

Applying KVL around the outer loop to get $V_{\text{Th}}$ across terminals $a$-$b$, we obtain

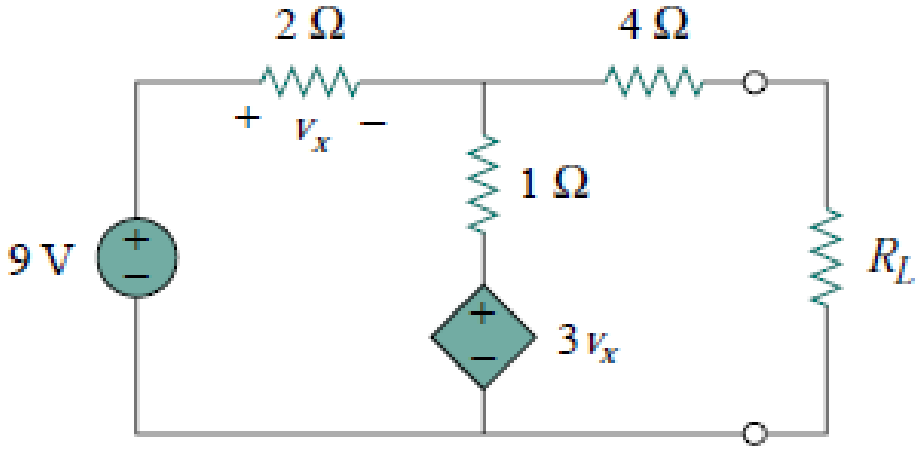
$$-12 + 6i_1 + 3i_2 + 2(0) + V_{\text{Th}} = 0 \quad \Longrightarrow \quad V_{\text{Th}} = 22\ \text{V}$$

For maximum power transfer, $R_L = R_{\text{Th}} = 9\ \Omega$ and the maximum power is

$$p_{\max} = \frac{V_{\text{Th}}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\ \text{W}$$

## ❑ Maximum Power Transfer

**Practice Problem:** Find the value of $R_L$ *for maximum power transfer in the circuit below*. Find the maximum power?




$$R_L = 4.22\Omega$$

$$p_{\max} = 2.901W$$

# END OF CHAPTER 3

Dr. Yılmaz KALKAN