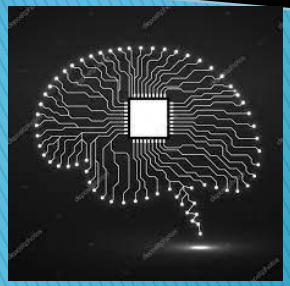
AYDIN
ADNAN MENDERES UNIVERSITY
FACULTY OF ENGINEERING
Department of Electrical and Electronics Engineering

EE201 – Circuit Theory I

Prof. Dr. Yılmaz KALKAN

circuit theory





1. **Basic Concepts (Chapter 1 of Nilsson – Week 1)**

   Introduction, Current and Voltage, Power and Energy

2. **Basic Laws (Chapter 2&3 of Nilsson – Week 2)**

   Voltage and Current Sources, Ohm's Law,

   Kirchhoff's Laws, Resistors in parallel and in series, Voltage and Current Division

3. **Techniques of Circuit Analysis (Chapter 4 of Nilsson – Weeks 3-6)**

   Node Analysis, Node-Voltage Method and Dependent Sources, Mesh Analysis, Mesh-Current Method and Dependent Sources, Source Transformations, Thevenin and Norton Equivalents, Maximum Power Transfer, Superposition Theorem

   MIDTERM (Weeks 7&8)

4. **Operational Amplifier (Chapter 5 of Nilsson – Weeks 9-10)**

   Op-Amp Terminals & Ideal Op-Amp, Basic Op-Amp Circuits, Buffer circuit, Inverting and Non-inverting Amplifiers, Summing Inverter, Difference Amplifier, Cascade Op-Amp Circuits

5. **Capacitors and Inductors (Chapter 6 of Nilsson – Weeks 11)**

   Inductors, Capacitors, Series and Parallel Combinations of them

6. **First Order Circuits (Chapter 7 of Nilsson – Weeks 12 & 13)**

   The Natural Response of an RL & RC Circuits, The Step Response of RL and RC Circuits, A General Solution for Step and Natural Responses, Integrating Amplifier Circuit

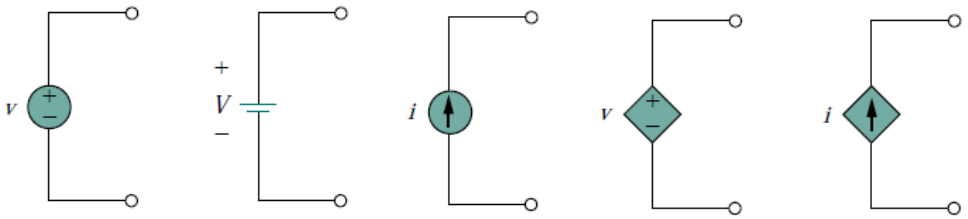
7. **Second Order Circuits (Chapter 8 of Nilsson – Weeks 14 & 15)**

   The Natural Response of a Parallel RLC Circuit, The Forms of Natural Response of a Parallel RLC Circuit, The Step Response of a Parallel RLC Circuit, Natural and Step Responses of a Series RLC Circuit

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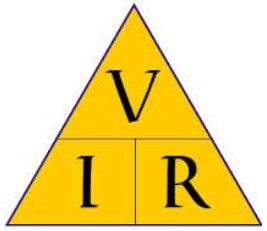
Summary of Chapter 2

- **Sources**
  - Voltage/Current
  - AC/DC
  - Dependent/Independent



- **Ohm's Law**
  - Resistors

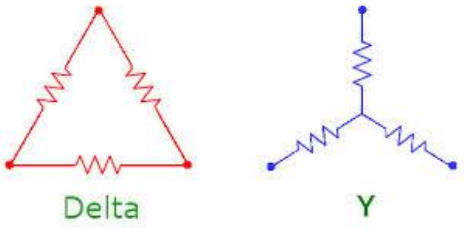
$$v = i.R$$



- **Nodes, Branches, Loops**
- **Kirchhoff's Laws**
  - Kirchhoff's Votage Law (KVL)
  - Kirchhoff's Current Law (KCL)

$$\sum_{n=1}^{N} i_n = 0 \qquad \sum_{m=1}^{M} v_m = 0$$

- **Series & Parallel Connections of Resistors**
  - Delta-to-Wye Transform
  - Current & Voltage Division



Ch3: Techniques of Circuit Analysis

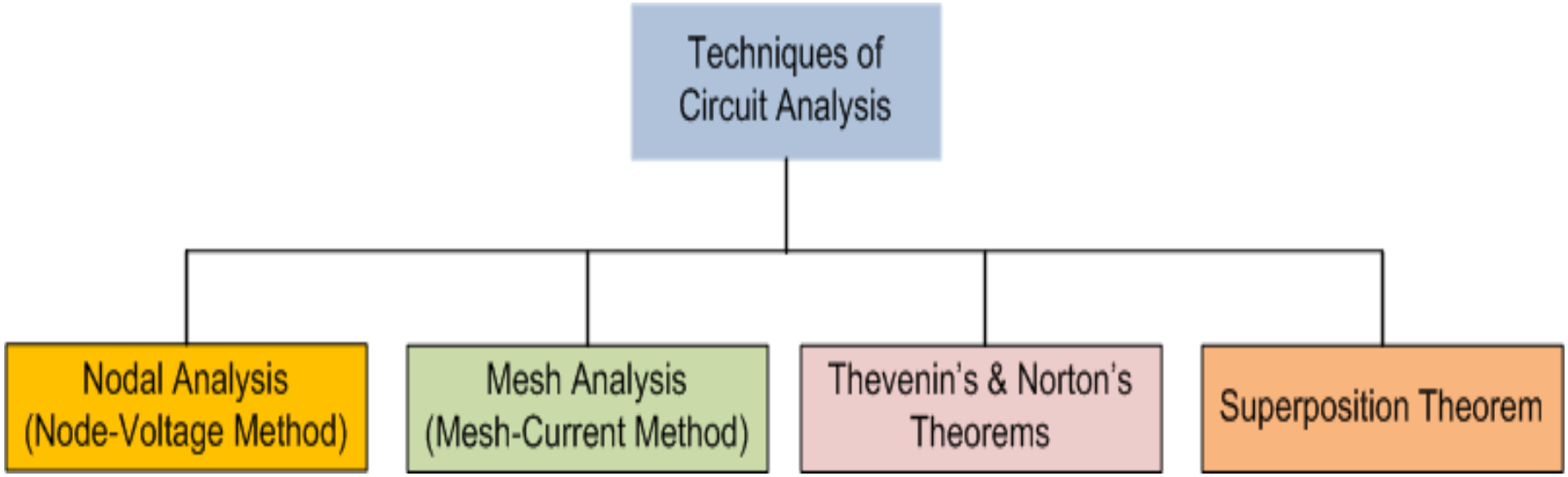
- Having understood the fundamental laws of circuit theory,
  Ohm's law
  Kirchhoff's laws (KVL & KCL)

- Apply these laws to develop two powerful techniques for circuit analysis.

- **Nodal analysis**, which is based on a systematic application of Kirchhoff's current law (**KCL**)

- **Mesh analysis**, which is based on a systematic application of Kirchhoff's voltage law (**KVL**).

Ch3: Techniques of Circuit Analysis



- **Source Transformations**
- **Maximum Power Transfer**

❑Nodal Analysis (Node-Voltage Method)

❑ **Nodal analysis** provides a general procedure for analyzing circuits using **node voltages as the circuit variables**. Choosing node voltages instead of element voltages as circuit variables is convenient and **reduces the number of equations** one must solve simultaneously.

❑ To simplify matters, we will assume firstly that circuits do not contain voltage sources. Circuits that contain voltage sources will be analyzed later.

❑ In nodal analysis, we are interested in **finding the node voltages**. Given a circuit with **N nodes** without voltage sources, the nodal analysis of the circuit involves taking the following three steps.

❑Nodal Analysis (Node-Voltage Method)

Steps to Determine Node Voltages:

1. Select a node as the **reference node**. Assign voltages $v_1, v_2, \ldots\ldots, v_{N-1}$ *to the remaining N − 1 nodes. The voltages are* referenced with respect to the reference node.

2. Apply KCL to each of the *N − 1* **nonreference nodes**. *Use Ohm's* law to express the branch currents in terms of node voltages.

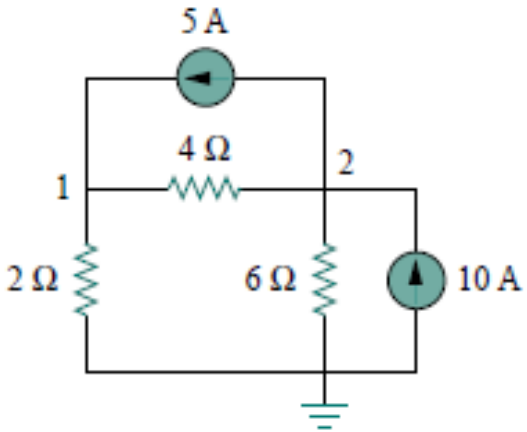
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

❑Nodal Analysis (Node-Voltage Method)

Steps to Determine Node Voltages:

1. Select a node as the **reference node**. Assign voltages $v_1, v_2, \ldots\ldots, v_{N-1}$ *to the remaining N − 1 nodes. The voltages are* referenced with respect to the reference node.

   ❑The reference node is commonly called the *ground since* it is assumed to **have zero potential.**

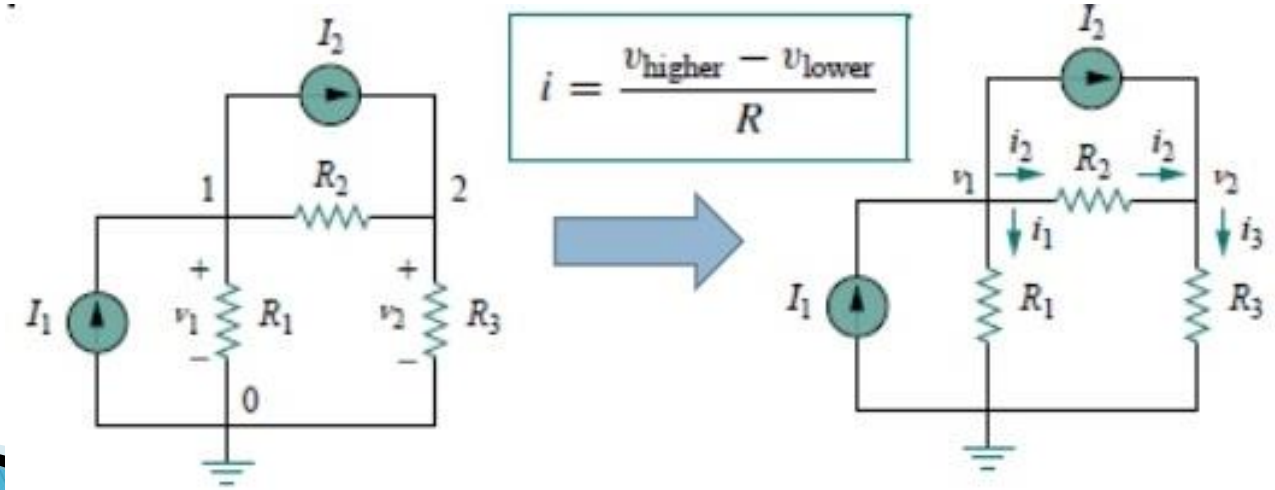


❑Nodal Analysis (Node-Voltage Method)

Steps to Determine Node Voltages:

2. Apply KCL to each of the *N − 1* **nonreference nodes**. *Use Ohm's* law to express the branch currents in terms of node voltages.

   ❑ Current flows from a **higher potential to a lower potential** in a resistor.



❑Nodal Analysis (Node-Voltage Method)

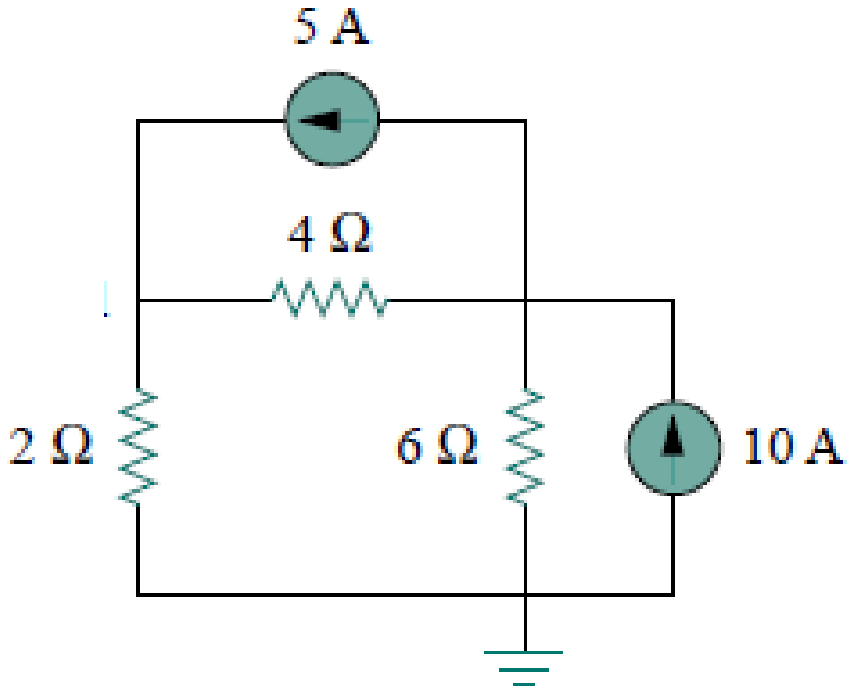
Steps to Determine Node Voltages:

3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

   ❑ Number of nonreference nodes is equal to the number of independent equations that we will derive.

❑Nodal Analysis (Node-Voltage Method)

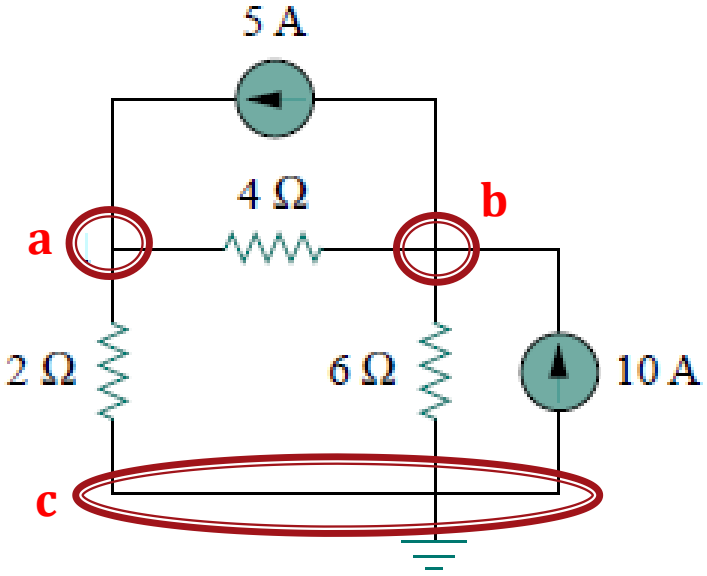
Example: Calculate the node voltages in the circuit given?



Steps:

1. Choose a reference node, assign voltages to other nodes w.r.t. reference one.
2. Apply KCL to each node. (Arbitrary but Consistent). Apply Ohm's law to find node voltages.
3. Solve all obtained equations together.
    1. Substitution method
    2. Elimination method
    3. Cramer's rule
    4. Matrix inversion
    5. …..

Example: Calculate the node voltages in the circuit given?



Steps:

1. Choose a reference node, assign voltages to other nodes w.r.t. reference one.

✓ We have 3 nodes. Node a, node b and node c.

✓ Choose any of them as reference node.

✓ Choose node c as reference node (ground).

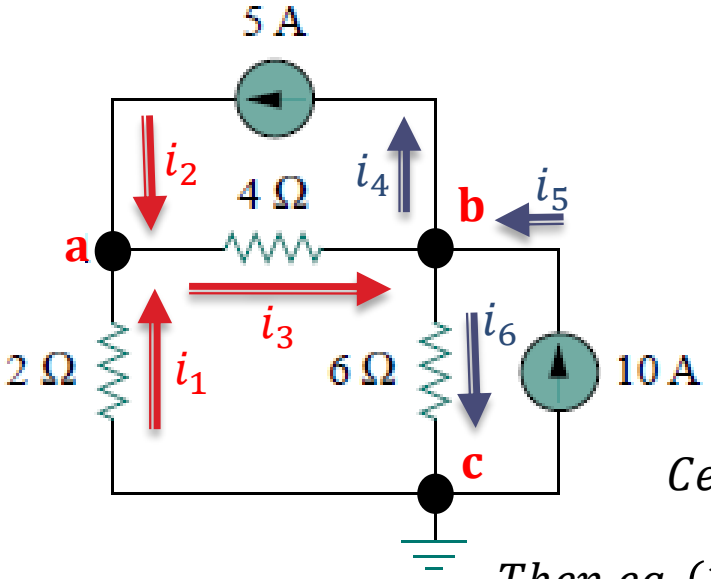
✓ Then, $V_c = 0$

✓ Assign voltages to other nodes w.r.t. reference one.

✓ Other nodes are node a and node b.

✓ Node voltages are; $V_a = V_{ac}$, $V_b = V_{bc}$

Example: Calculate the node voltages in the circuit given?



Steps:

2. Apply KCL to each node. (Arbitrary but Consistent).

Apply Ohm's law to find node voltages.

$$KCL\ @node\ a \rightarrow i_1 + i_2 = i_3 \quad \ldots\ldots\ldots..(1)$$

$$KCL\ @node\ b \rightarrow i_3 + i_5 = i_4 + i_6 \quad \ldots\ldots(2)$$

$$Certainly;\ i_2 = i_4 = 5A \rightarrow \text{Similarly};\ i_5 = 10A$$

$$Then\ eq.(1)\ becomes \rightarrow i_1 + 5 = i_3 \rightarrow i_1 - i_3 = -5 \quad \ldots\ldots(3)$$

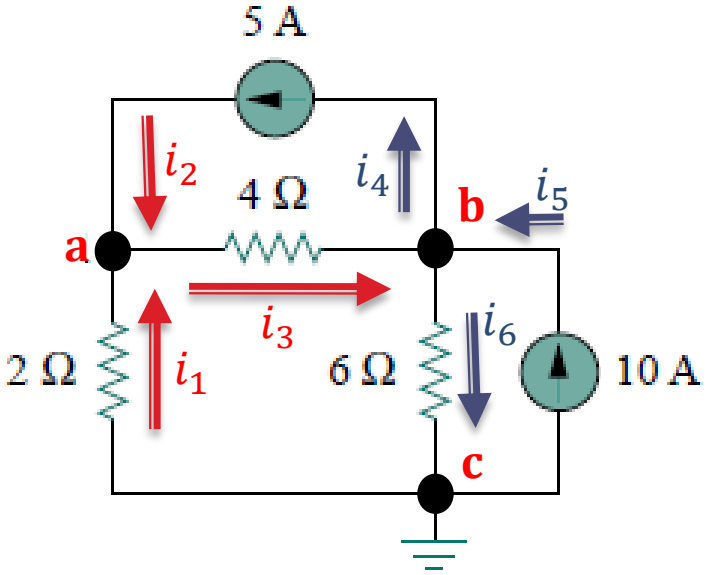
$$And\ eq.(2)\ becomes \rightarrow i_3 + 10 = 5 + i_6 \rightarrow i_3 - i_6 = -5 \quad \ldots\ldots(4)$$

Apply Ohm's law to represent currents with node voltages. $i = {}^{V}/_{R}$

Unknowns are; i_1, i_3 and i_6

$$i_1 = \frac{V_{2\Omega}}{2\Omega} = \frac{V_c - V_a}{2} = \frac{-V_a}{2} \qquad i_3 = \frac{V_a - V_b}{4} \qquad i_6 = \frac{V_b - V_c}{6} = \frac{V_b}{6}$$

Example: Calculate the node voltages in the circuit given?



Steps:

3. Solve all obtained equations together. (Substitution method, Elimination method, Cramer's rule, Matrix inversion, …..)

$$KCL\ @node\ a \rightarrow i_1 - i_3 = -5 \ \ldots\ldots (3)$$

$$KCL\ @node\ b \rightarrow i_3 - i_6 = -5 \ \ldots\ldots (4)$$

$$i_1 = \frac{-V_a}{2} \qquad i_3 = \frac{V_a - V_b}{4} \qquad i_6 = \frac{V_b}{6}$$

Insert currents into KCL equations.

$Unknowns\ are\ NOW;$ $V_a\ and\ V_b$

$$\rightarrow i_1 - i_3 = -5 \rightarrow \frac{-V_a}{2} - \frac{V_a - V_b}{4} = -5 \rightarrow V_b - 3V_a = -20 \rightarrow 3V_a - V_b = 20$$

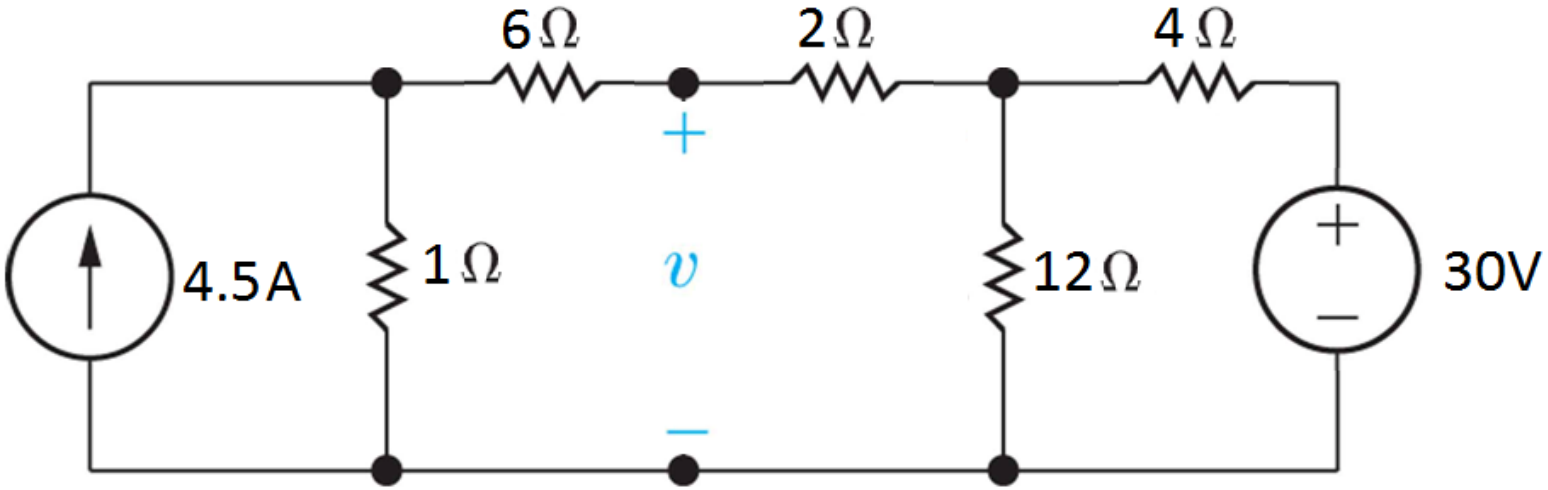
$$\rightarrow i_3 - i_6 = -5 \rightarrow \frac{V_a - V_b}{4} - \frac{V_b}{6} = -5 \rightarrow 3V_a - 5V_b = -60$$

$$V_a = {}^{40}/_{3}\ V$$

$$V_b = 20\ V$$

❑Nodal Analysis (Node-Voltage Method)

Example: Use the node-voltage method to find v in the circuit shown? (Assesment problem 4.2 from textbook)

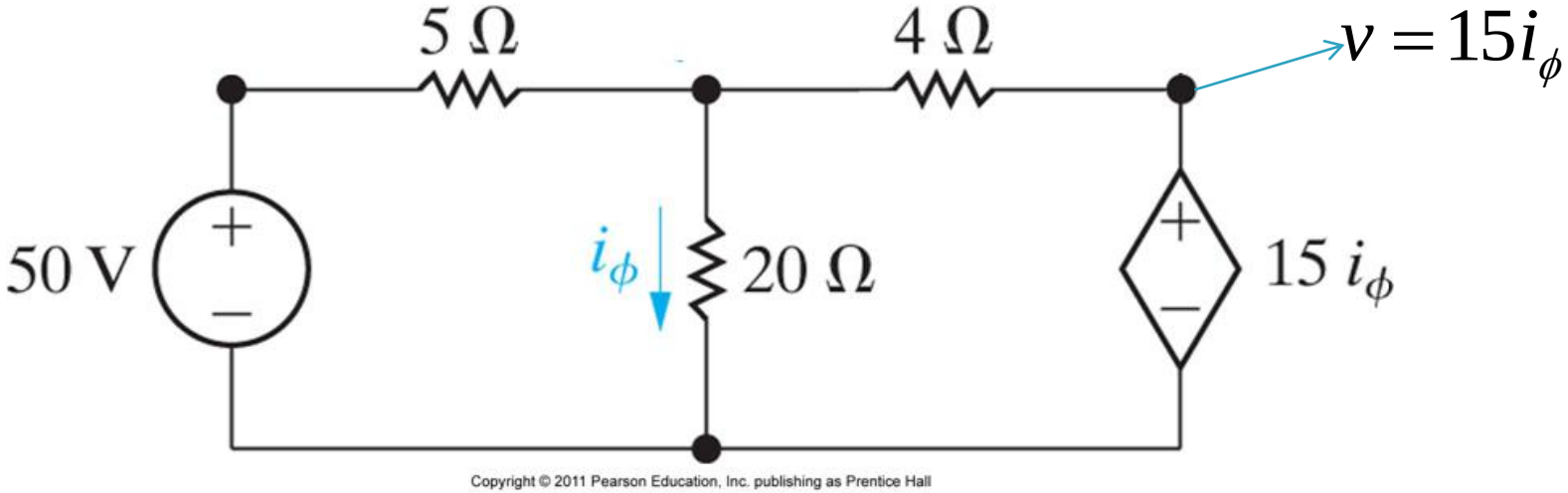


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Answer : 15 V

❑ Nodal Analysis (Node-Voltage Method) with Dependent Sources

❑ The node-voltage equations must be supplemented with the **constraint equations** imposed by the presence of the dependent sources.

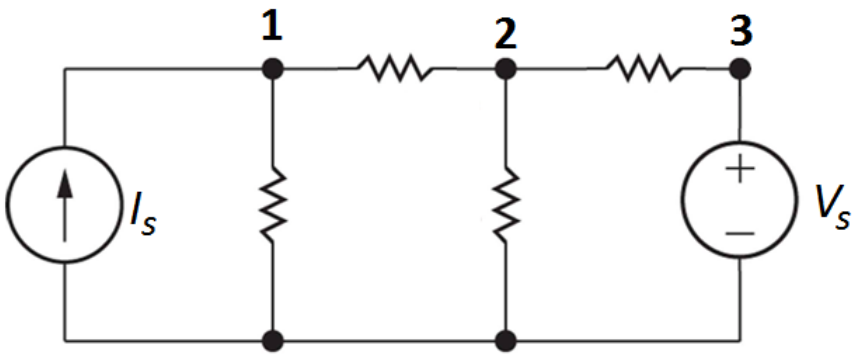


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Constraint equation : $i_\phi = \dfrac{v_{20\Omega}}{20}$

❑ Modified Nodal Analysis (Modified Node-Voltage Method)

- ❑ When a voltage source is directly connected in between a reference node and a non-reference node.
  - ❑ The voltage of that non-reference node = voltage of that source.
  - ❑ Hence, the number of required equations is decreased by one.

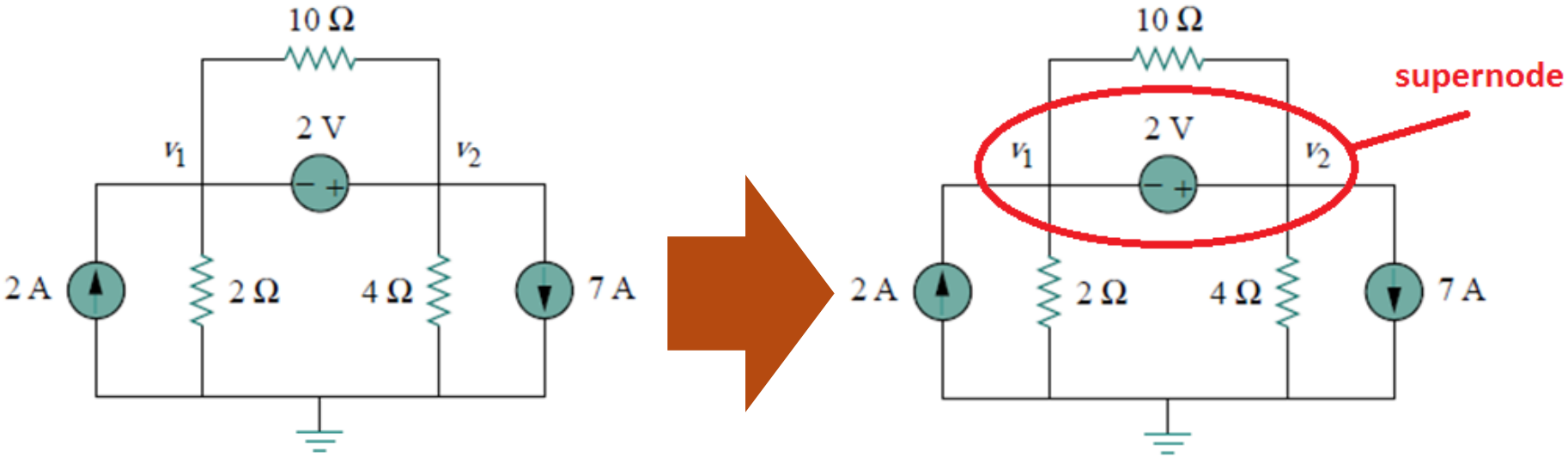


$$v_3 = v_s$$

❑ Modified Nodal Analysis (Modified Node-Voltage Method)

❑ When a voltage source is the only element between two essential nodes (or non-reference nodes), the node-voltage method is simplified.

❑ For this case, those nodes can be combined to form a **SUPERNODE**.



❑ Modified Nodal Analysis (Modified Node-Voltage Method)

Note the following properties of a supernode:

1. The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
2. A supernode has no voltage of its own.
3. A supernode requires the application of both KCL and KVL.

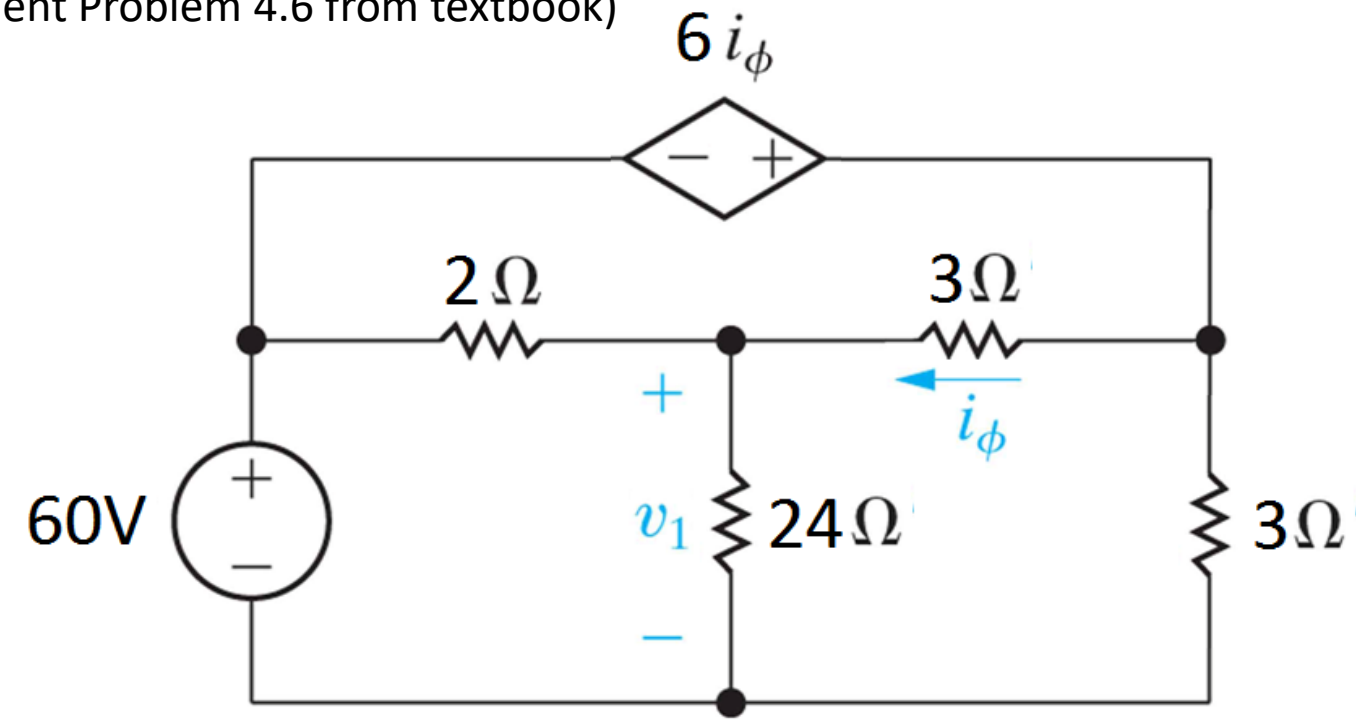
Steps for Supernode case:

1. Assign a current for that branches.
2. Use these currents as the additional variables.
3. For supernode, use KCL to obtain supernode equation.

To obtain Node-voltage equations, nodes can be used separately or used as a supernode (if available). Both gives the same result but using supernode is decreased the number of equations.

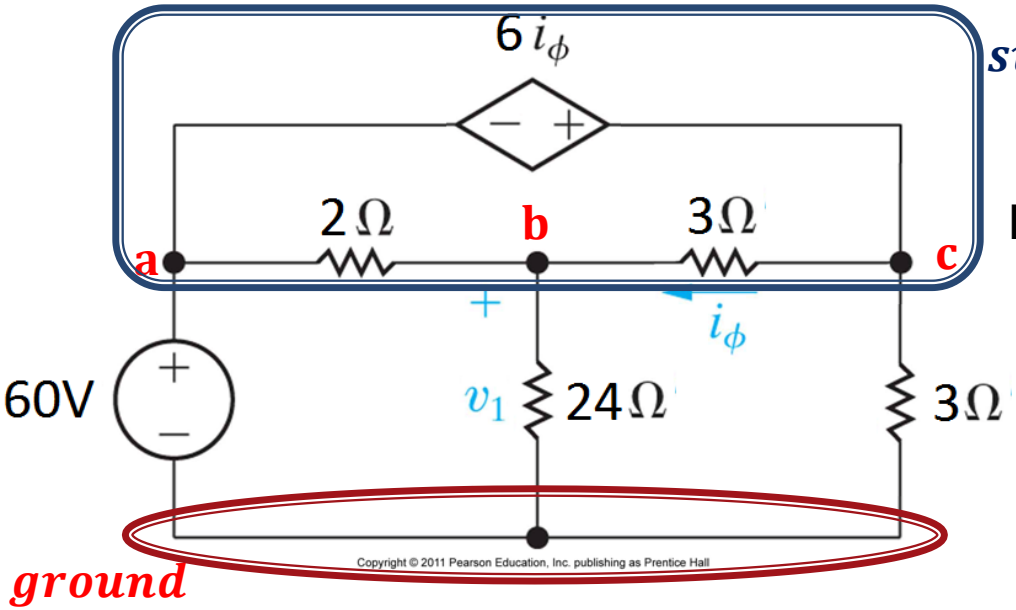
❑ Modified Nodal Analysis (Modified Node-Voltage Method)

Example: Use the node-voltage method to find v_1 *in the* circuit shown?
(Assesment Problem 4.6 from textbook)



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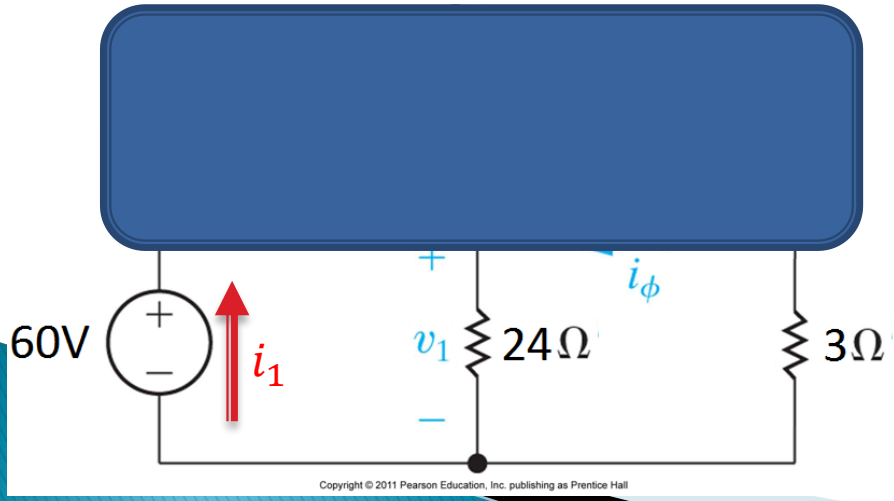
Example: Use the node-voltage method to find v_1 *in the* circuit shown?



super node

ground

But we couldn't use supernode here. Why?

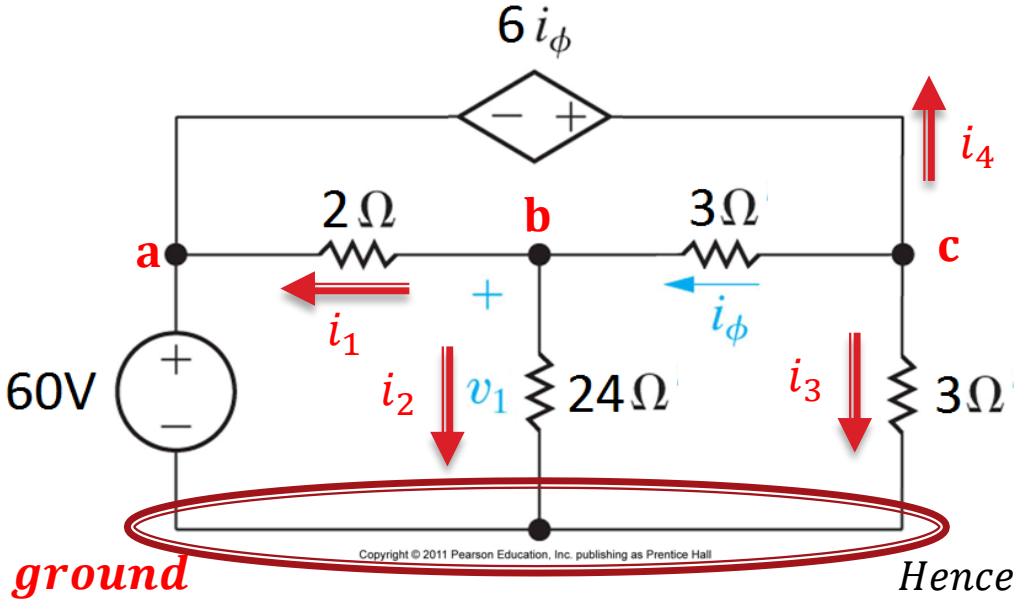


To write KCL for supernode; you need to define i_1
What is i_1?

We can not write i_1 in terms of node voltages. Hence, SuperNode can not be used here.

Ch3: Techniques of Circuit Analysis

Example: Use the node-voltage method to find v_1 *in the* circuit shown?



ground

$V_a = 60V$

$V_b = v_1$

$V_c = 60 + 6i_\varphi$

Unknowns are V_b *and* V_c

Hence we need to write KCL @ node a and @node b.

$$KCL\ @node\ b \rightarrow i_1 + i_2 = i_\varphi \rightarrow \frac{V_b - V_a}{2} + \frac{V_b - 0}{24} = \frac{V_c - V_b}{3} \rightarrow 21V_b - 8V_c = 12V_a$$

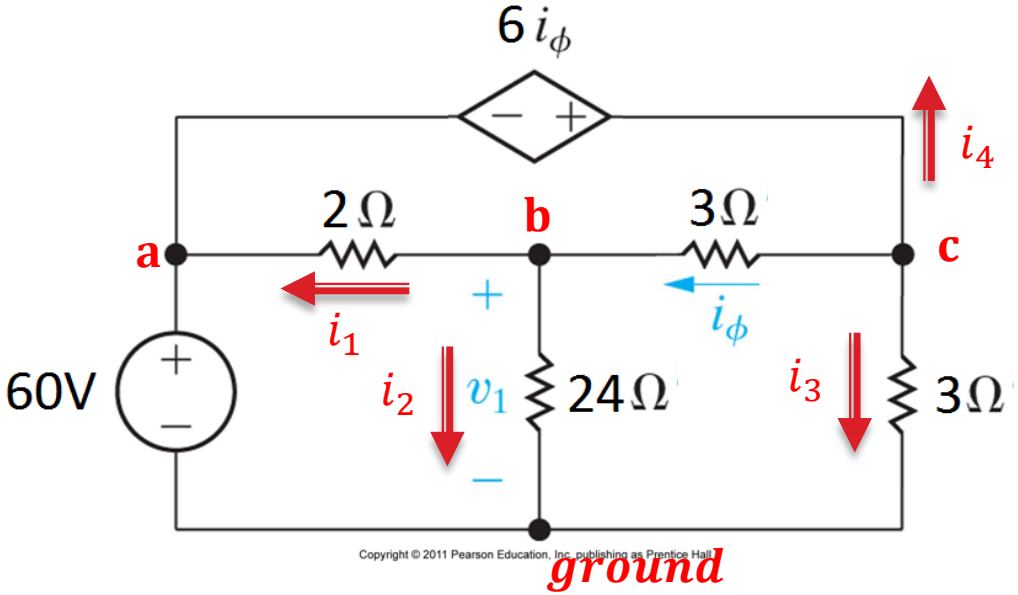
$$\rightarrow 21V_b - 8V_c = 720 \ldots (1)$$

$$KCL\ @node\ c \rightarrow i_3 + i_4 + i_\varphi = 0$$

we have same problem for i_4!!
Hence we couldn't use this KCL equation!!

Ch3: Techniques of Circuit Analysis

Example: Use the node-voltage method to find v_1 *in the* circuit shown?



ground

$V_a = 60V$

$V_b = v_1$

$V_c = 60 + 6i_\varphi$

Unknowns are V_b *and* V_c

$\rightarrow 21V_b - 8V_c = 720 \quad \ldots(1)$

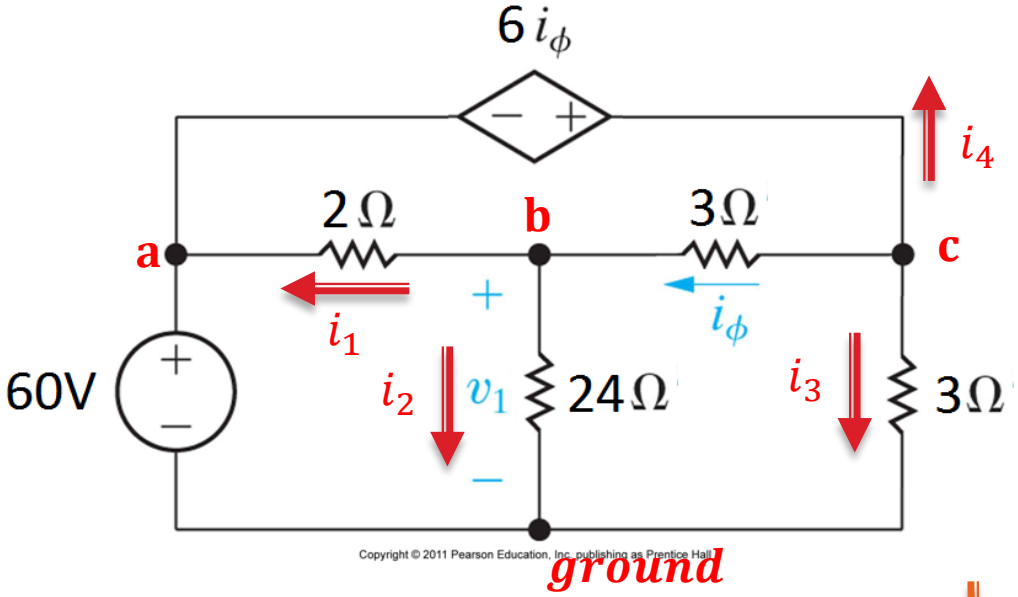
We have one equation and two unknowns. To find unknowns, we need the 2nd equation with same unknowns. What is it?

We have a dependent source. It means ve have another equation from there.

$i_\varphi = ? \rightarrow i_\varphi = \dfrac{V_c - V_b}{3}$ insert this one into $V_c = 60 + 6i_\varphi$

Then we have; $V_c = 60 + 6\dfrac{V_c - V_b}{3} \rightarrow 2V_b - V_c = 60 \quad \ldots(2)$

Example: Use the node-voltage method to find v_1 *in the* circuit shown?



$V_b = v_1$

Unknowns are V_b *and* V_c

$21V_b - 8V_c = 720$ ... (1)

$2V_b - V_c = 60$ ... (2)

$$21V_b - 8V_c = 720$$

$$-8x/\ 2V_b - V_c = 60$$

$$+\ \overline{21V_b - 8V_c = 720}$$

$$+\ \overline{-16V_b + 8V_c = -480}$$

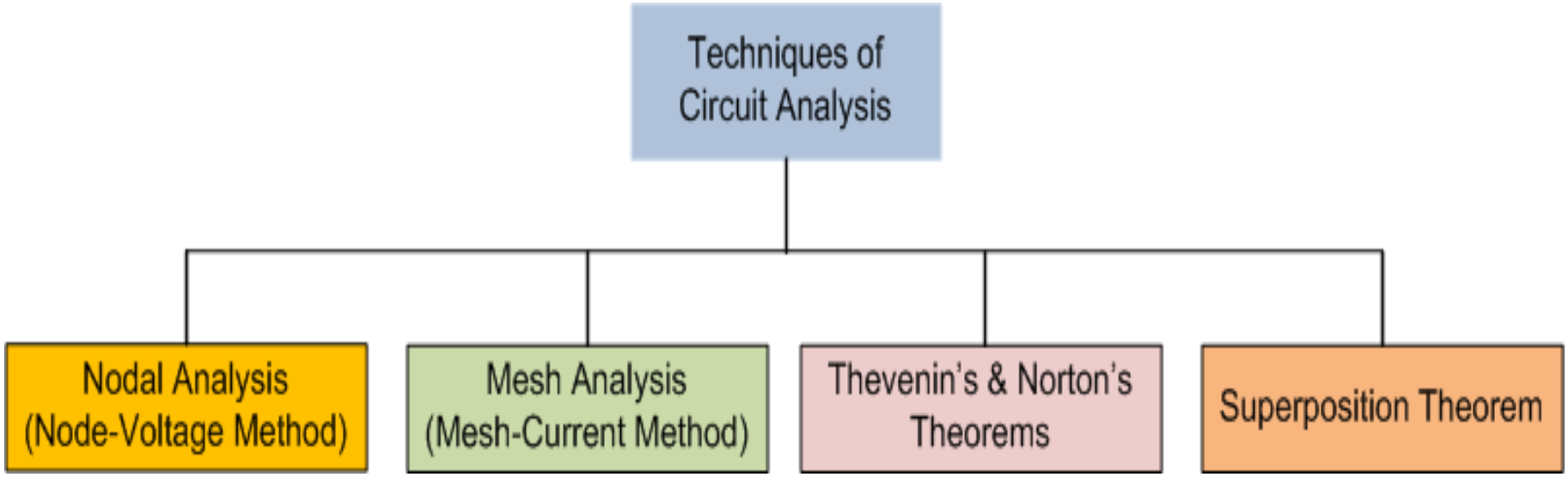
$$5V_b = 240$$

$$V_b = 48\ V$$

$$v_1 = V_b = 48\ V$$

Ch3: Techniques of Circuit Analysis

Summary of Last Week



- ❑Source Transformations
- ❑Maximum Power Transfer

❑Nodal Analysis (Node-Voltage Method)

Steps to Determine Node Voltages:

1. Select a node as the **reference node**. Assign voltages $v_1, v_2, \ldots\ldots, v_{N-1}$ *to the remaining N − 1 nodes. The voltages are* referenced with respect to the reference node.

2. Apply KCL to each of the *N − 1* **nonreference nodes**. *Use Ohm's* law to express the branch currents in terms of node voltages.

3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

❑ Mesh Analysis (Mesh-Current Method)

❑ **Mesh**: A loop does not enclose any other loops.

❑ Nodal analysis applies KCL to find unknown voltages in a given circuit, while, **mesh analysis** applies **KVL** to find unknown currents.

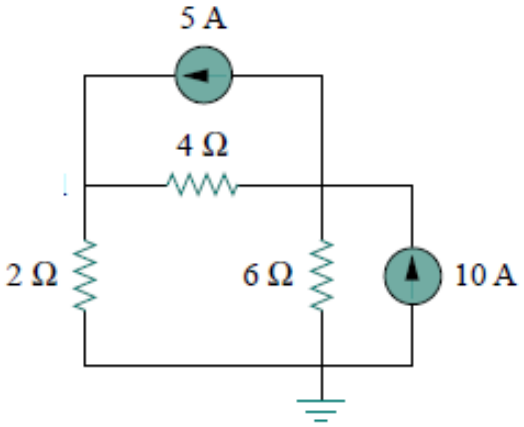
❑ Nodal analysis can be applied all circuits in general. On the other hand, Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is **planar**.

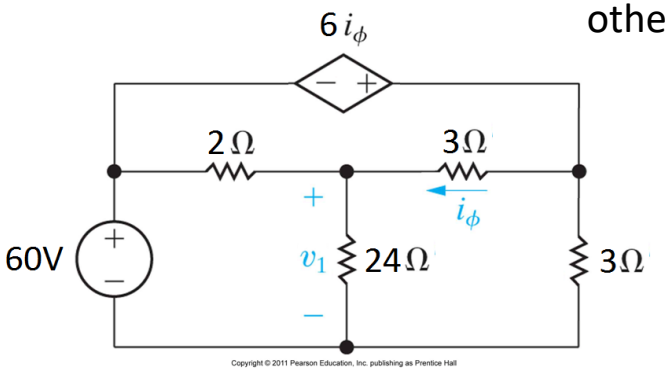
❑ A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is **nonplanar**.

❑ A circuit may have crossing branches and still be planar if it can be redrawn such that it has no crossing branches.

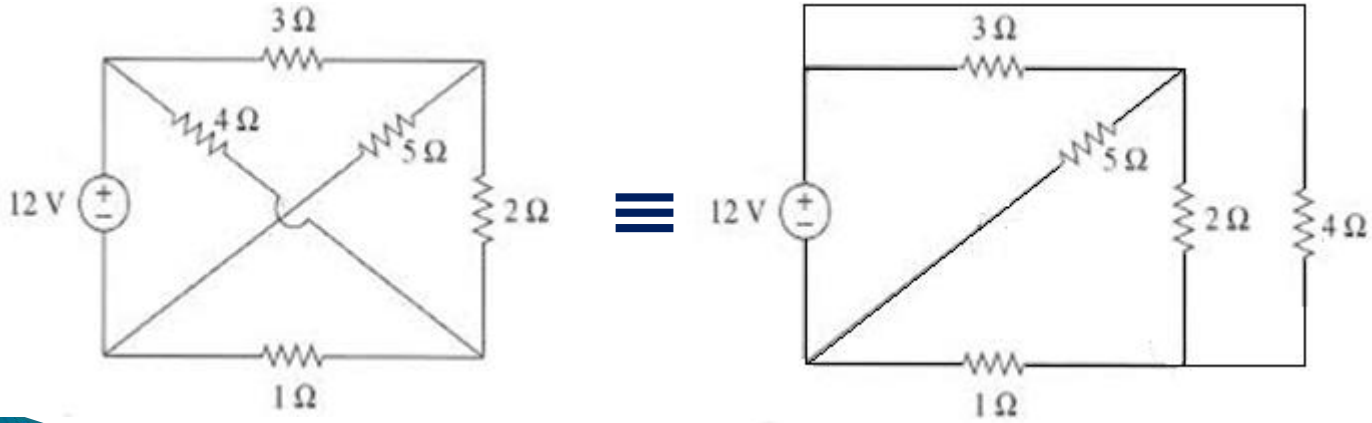
❑ Mesh Analysis (Mesh-Current Method)

❑A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is **nonplanar**.





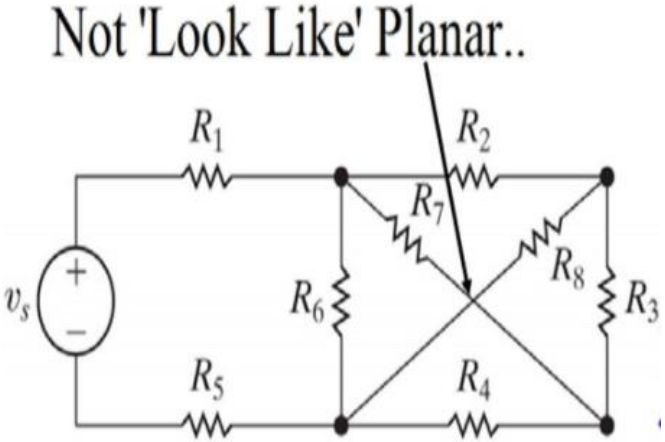
Planar circuits

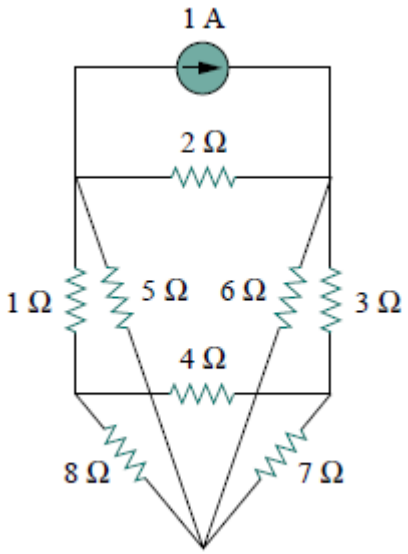


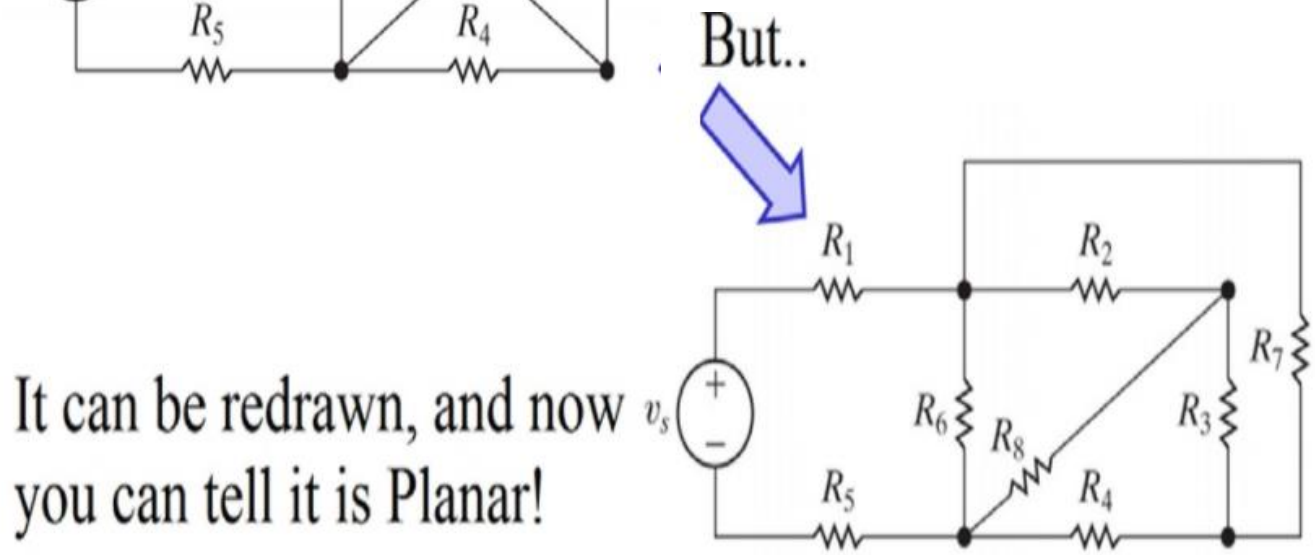
Planar circuit

❑ Mesh Analysis (Mesh-Current Method)

❑A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is **nonplanar**.

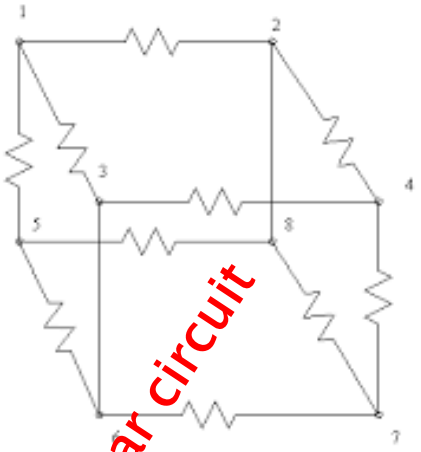






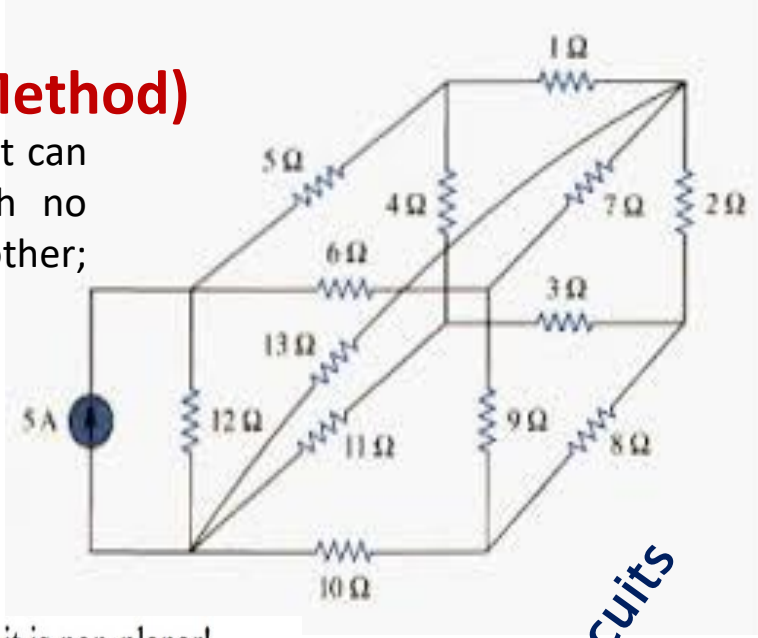
All Planar circuits

❑ Mesh Analysis (Mesh-Current Method)

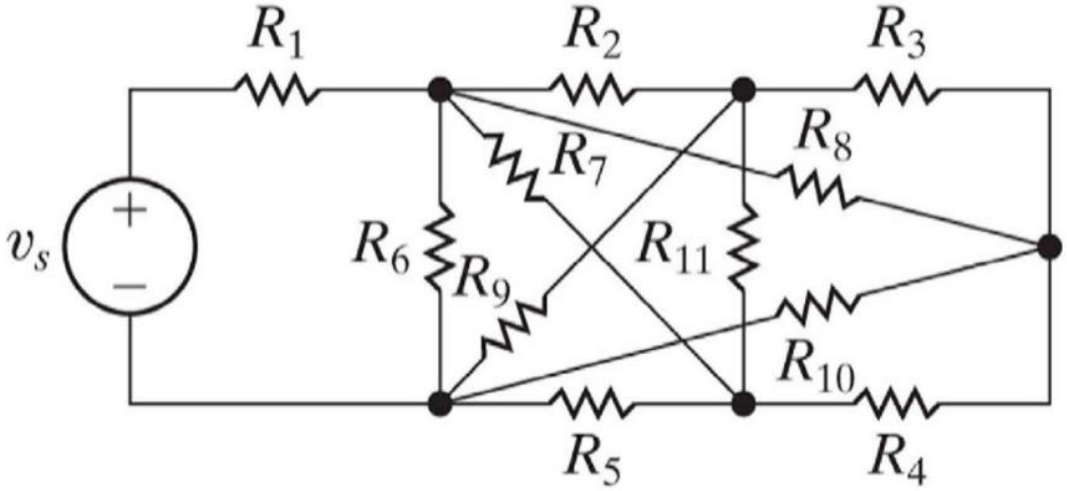


Planar circuit

❑A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is **nonplanar**.



Cannot be redrawn into a plnar circuit, therefore it is non-planar!



NOT Planar circuits

❑ Mesh Analysis (Mesh-Current Method)

❑ The current through a mesh is known as mesh current. In mesh analysis, we are interested in applying KVL to find the mesh currents in a given circuit.

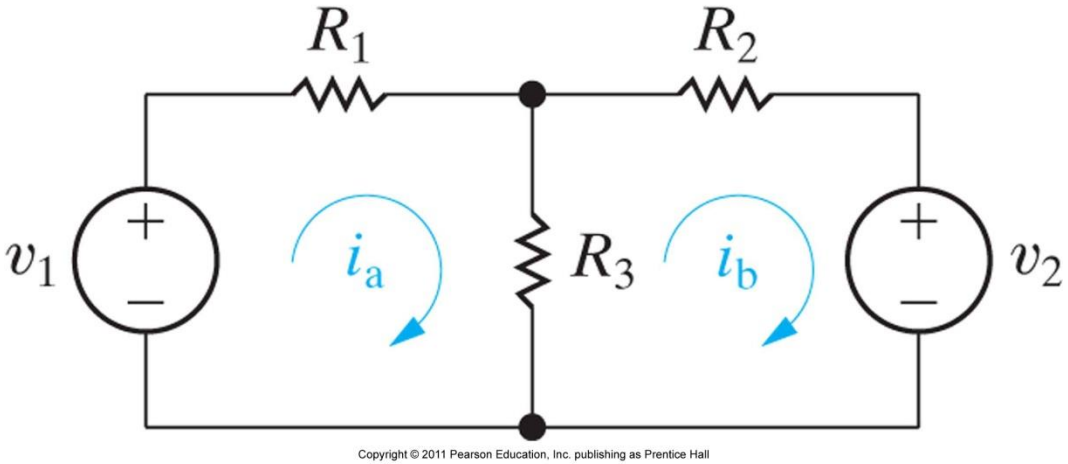
Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots\ldots, i_{N-1}$ to the N meshes.
2. Apply KVL to each of the *N* meshes. *Use Ohm's* law to express the voltages in terms of the mesh currents.
3. Solve the resulting N simultaneous equations to get the unknown mesh currents.

❑ Mesh Analysis (Mesh-Current Method)

Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots\ldots, i_{N-1}$ to the N meshes.



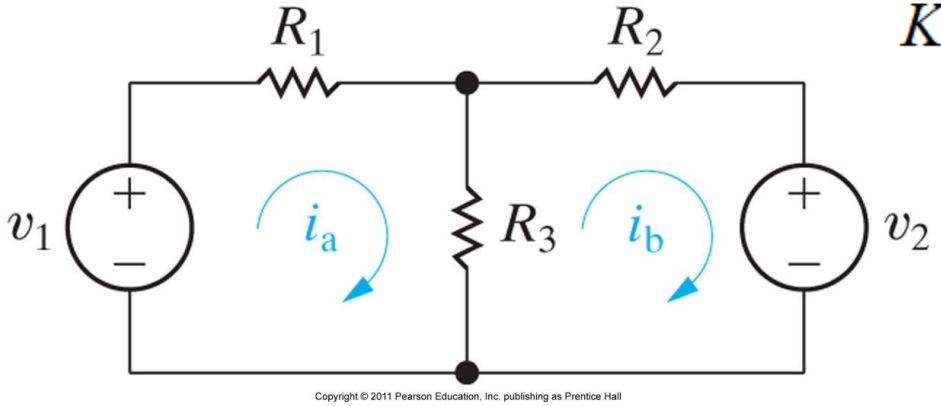
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Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.

❑ Mesh Analysis (Mesh-Current Method)

Steps to Determine Mesh Currents:

2. Apply KVL to each of the *N* meshes. *Use Ohm's* law to express the voltages in terms of the mesh currents.



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$$KVL\ @\ mesh\ a \Rightarrow -v_1 + v_{R_1} + v_{R_3} = 0$$

$$KVL\ @\ mesh\ b \Rightarrow \quad v_2 + v_{R_2} + v_{R_3} = 0$$

Ohm's Law

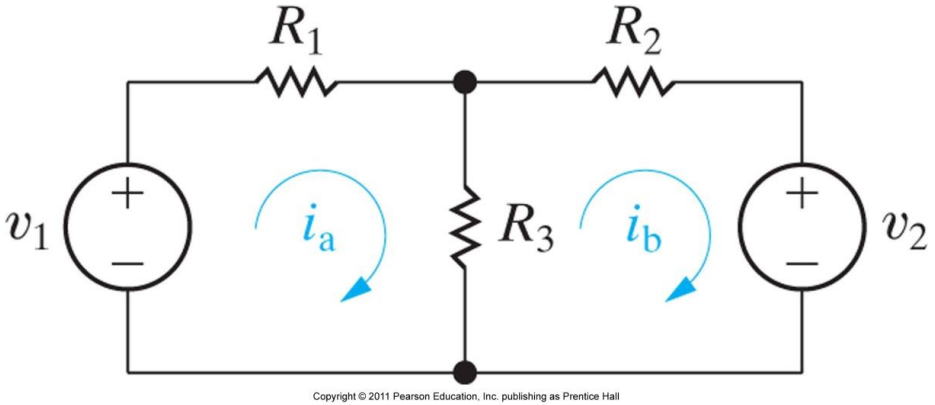
$$-v_1 + R_1 i_a + R_3(i_a - i_b) = 0$$

$$v_2 + R_2 i_b + R_3(i_b - i_a) = 0$$

❑ Mesh Analysis (Mesh-Current Method)

Steps to Determine Mesh Currents:

3. Solve the resulting N simultaneous equations to get the unknown mesh currents.



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$$-v_1 + R_1 i_a + R_3(i_a - i_b) = 0$$

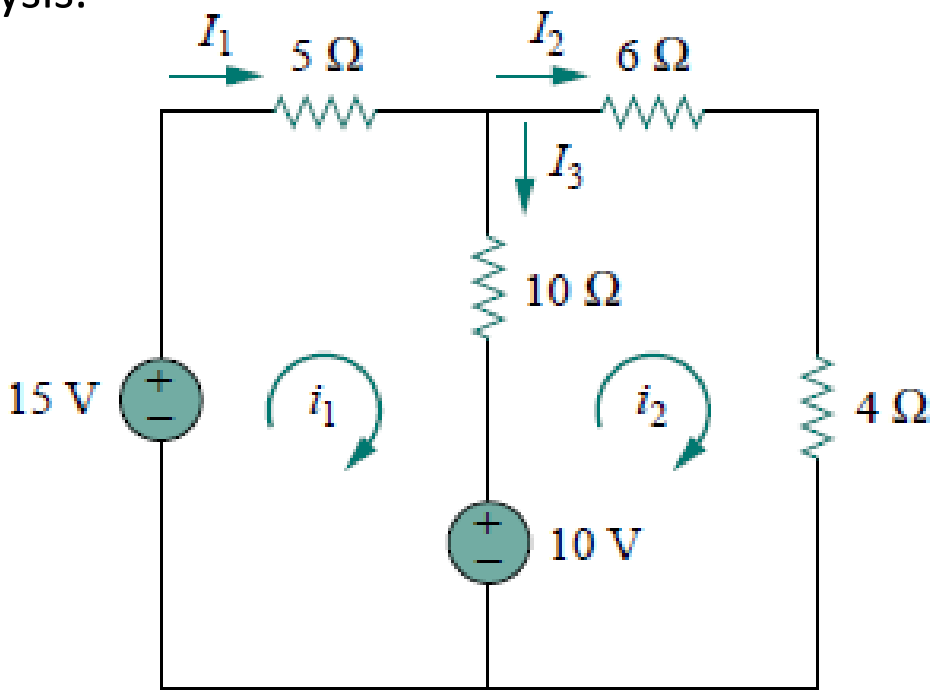
$$v_2 + R_2 i_b + R_3(i_b - i_a) = 0$$

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_1 + R_3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix} \Rightarrow i_a \ \& \ i_b$$

$$l = b - n + 1$$

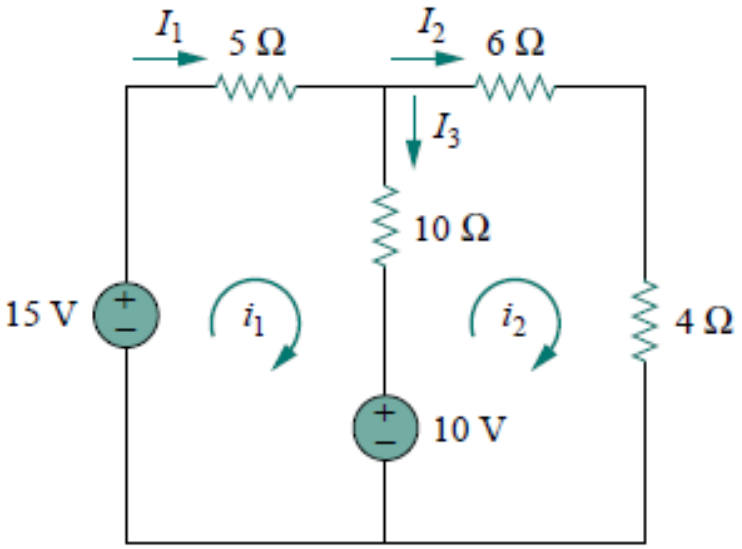
❑ Mesh Analysis (Mesh-Current Method)

Example : For the circuit given below, find the branch currents I_1, I_2 *and* I_3 u*sing* mesh analysis.



❑ Mesh Analysis (Mesh-Current Method)

Example : For the circuit given below, find the branch currents I_1, I_2 *and* I_3 u*sing* mesh analysis.



$I_1 = i_1$, $I_2 = i_2$, $I_3 = i_1 - i_2$

$$KVL\ @mesh\ 1 \rightarrow -15 + V_{5\Omega} + V_{10\Omega} + 10 = 0$$

$$KVL\ @mesh\ 2 \rightarrow -10 + V_{10\Omega} + V_{6\Omega} + V_{4\Omega} = 0$$

Ohm's Law (V=IR)

$$V_{5\Omega} = I_1 5 = 5I_1$$

$$V_{6\Omega} = I_2 6 = 6I_2 \quad , V_{4\Omega} = I_2 4 = 4I_2$$

$$V_{10\Omega} = I_3 10 = 10I_3 = 10(I_1 - I_2)$$

$$KVL\ @mesh\ 1 \rightarrow -15 + 5I_1 + 10(I_1 - I_2) + 10 = 0 \rightarrow 3I_1 - 2I_2 = 1$$

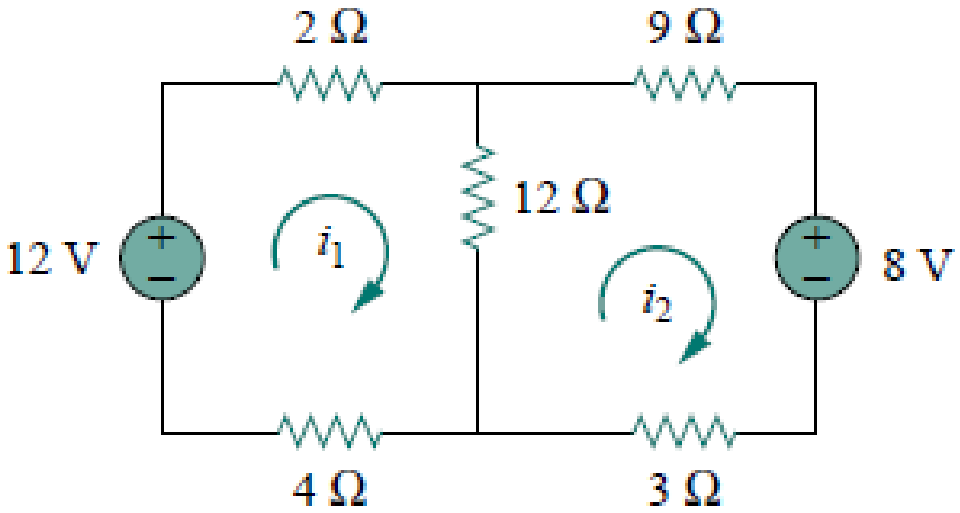
$$KVL\ @mesh\ 2 \rightarrow -10 + 10(I_2 - I_1) + 6I_2 + 4I_2 = 0 \rightarrow -I_1 + 2I_2 = 1$$

$$I_1 = I_2 = 1A$$

$$I_3 = 0$$

❑ Mesh Analysis (Mesh-Current Method)

Exercise : Calculate the mesh currents i_1 *and* i_2 *in the circuit below*.?

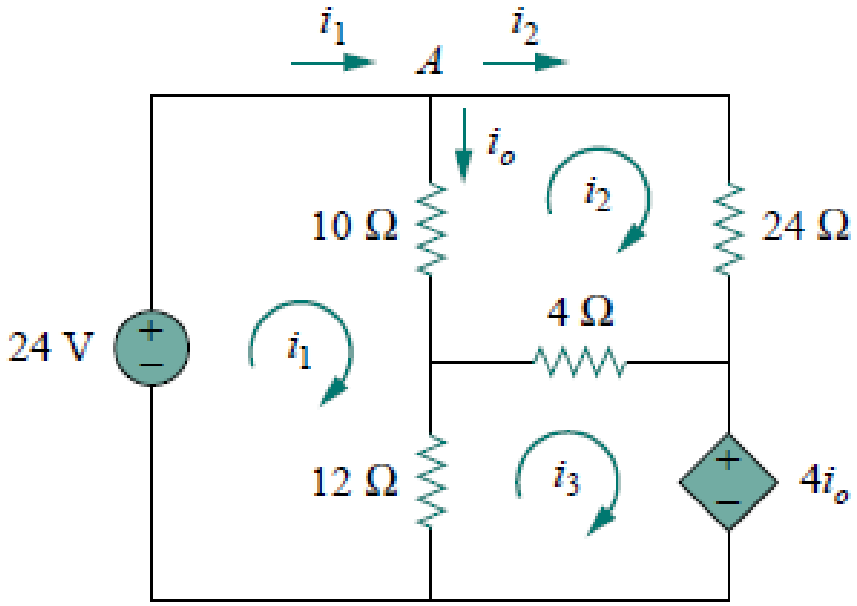


Answer: $i_1 = 2/3A,\ i_2 = 0$

❑ Mesh Analysis (Mesh-Current Method) with dependent sources

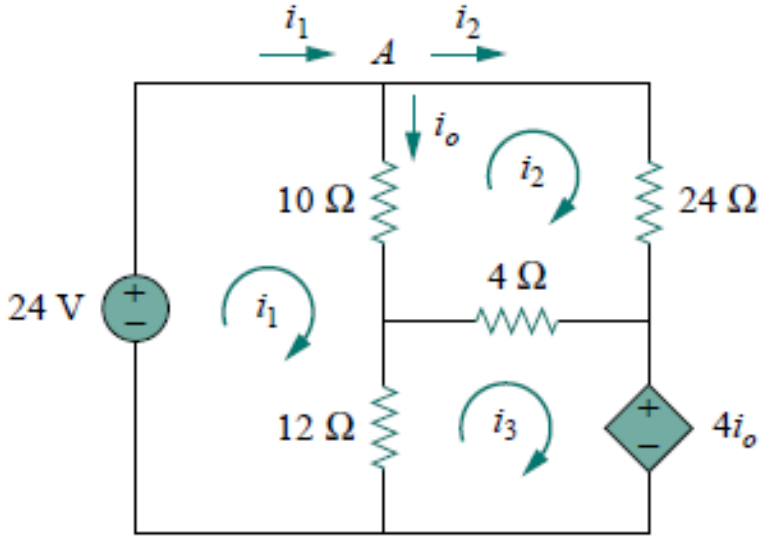
- ❑ Only difference is that we have an extra **constraint equation**.

Example: Use mesh analysis to find the current i_0 *in the circuit* below?



❑ Mesh Analysis (Mesh-Current Method) with dependent sources

Example: Use mesh analysis to find the current i_0 *in the circuit* below?



$$KVL\ @mesh\ 1 \rightarrow -24 + V_{10\Omega} + V_{12\Omega} = 0$$

$$KVL\ @mesh\ 2 \rightarrow V_{10\Omega} + V_{24\Omega} + V_{4\Omega} = 0$$

$$KVL\ @mesh\ 3 \rightarrow V_{12\Omega} + V_{4\Omega} + 4i_0 = 0$$

Apply Ohm's Law to represent Voltages in terms of mesh curents .

$$@mesh\ 1 \rightarrow 22i_1 - 10i_2 - 12i_3 = 24$$

$$@mesh\ 2 \rightarrow -10i_1 + 38i_2 - 4i_3 = 0$$

$$@mesh\ 3 \rightarrow -12i_1 - 4i_2 + 16i_3 = -4i_0$$

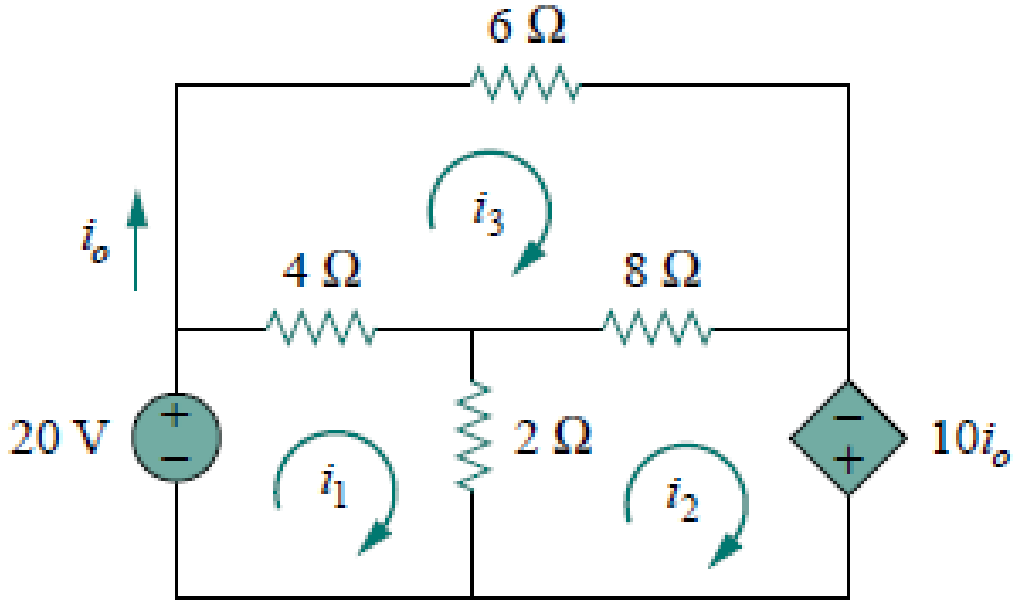
Answer: $i_0 = 1.5A$

$$Constraint\ equation:\ i_0 = i_1 - i_2 \longrightarrow -12i_1 - 4i_2 + 16i_3 = -4(i_1 - i_2)$$

$$-8i_1 - 8i_2 + 16i_3 = 0$$

❑ Mesh Analysis (Mesh-Current Method) with dependent sources

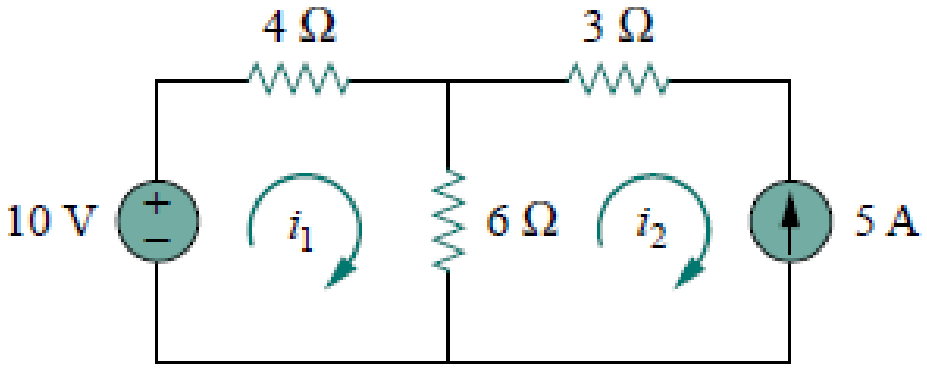
Exercise: Use mesh analysis to find the current i_0 *in the circuit* below?



Answer: $i_0 = -5A$

❑ Modified Mesh Analysis (Modified Mesh-Current Method)

- ❑ When a current source exist only in one mesh:
  - ❑ The current of that mesh = current of that source.
  - ❑ Hence, the number of required equations is decreased by one.

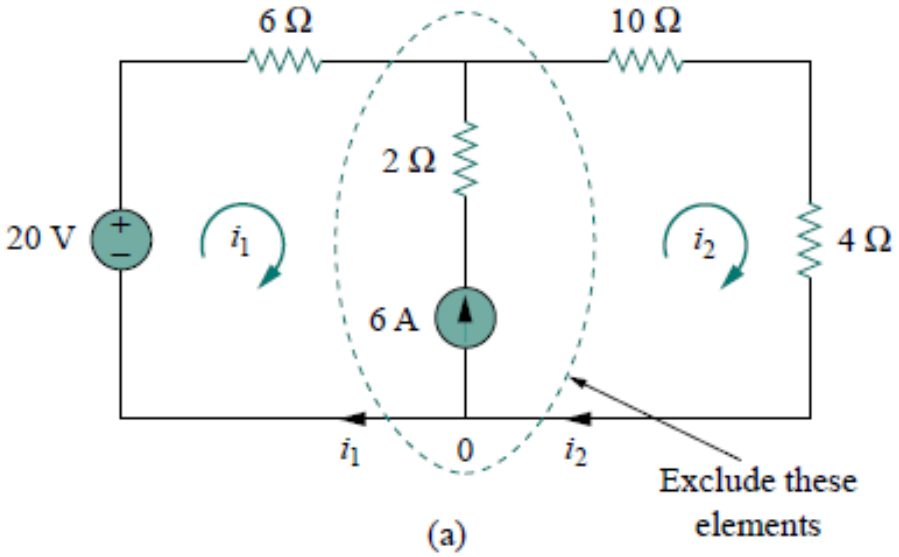


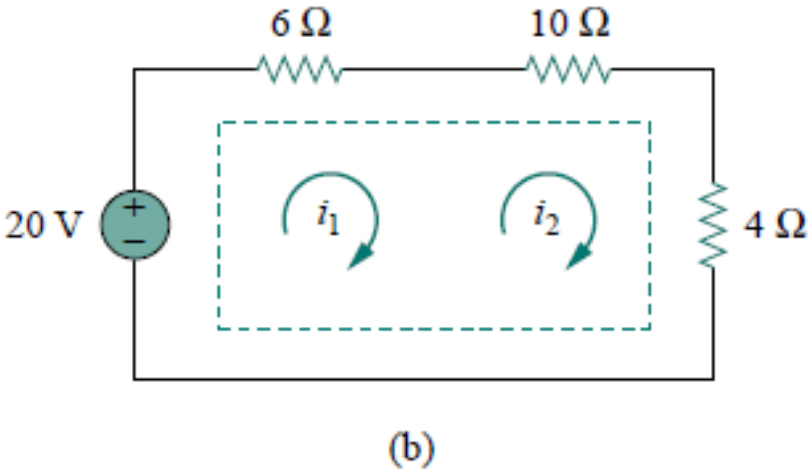
$$i_2 = -5A$$

❑ Modified Mesh Analysis (Modified Mesh-Current Method)

❑ When a current source exist between two meshes, the mesh-current method is simplified.

❑ For this case, those meshes can be combined to form a **SUPERMESH**.





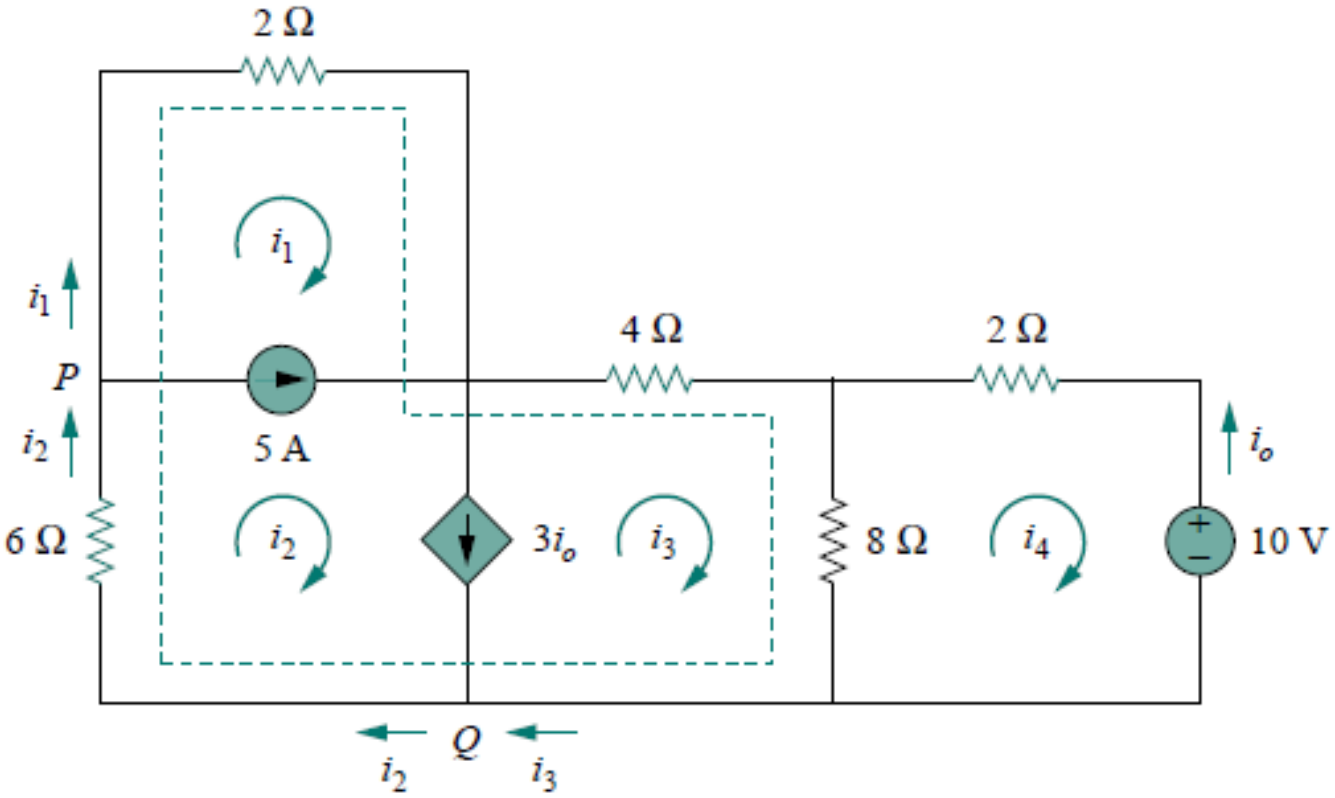
$$i_2 - i_1 = 6A \quad \&\& \quad 6i_1 + 14i_2 = 20$$

$$i_1 = -3.2A$$

$$i_2 = 2.8A$$

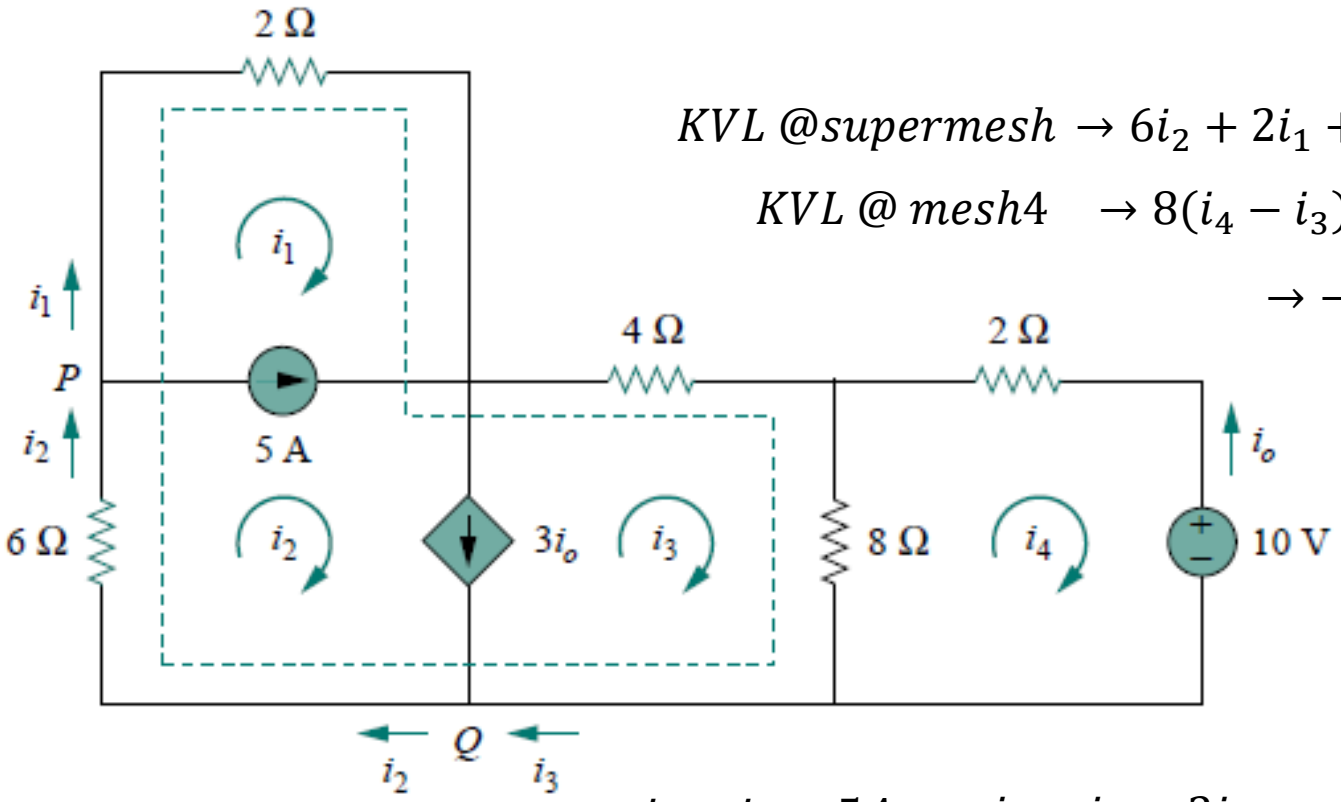
❑ Modified Mesh Analysis (Modified Mesh-Current Method)

Example: For the circuit given below, find all currents ***<u>using mesh analysis</u>***?



❑ Modified Mesh Analysis (Modified Mesh-Current Method)

Example: For the circuit given below, find all currents ***using mesh analysis***?



$$KVL\ @supermesh \rightarrow 6i_2 + 2i_1 + 12i_3 - 8i_4 = 0$$

$$KVL\ @\ mesh4 \rightarrow 8(i_4 - i_3) + 2i_4 + 10 = 0$$

$$\rightarrow -8i_3 + 10i_4 = -10$$

4 unknowns but we have only 2 equations??

What do we have else?

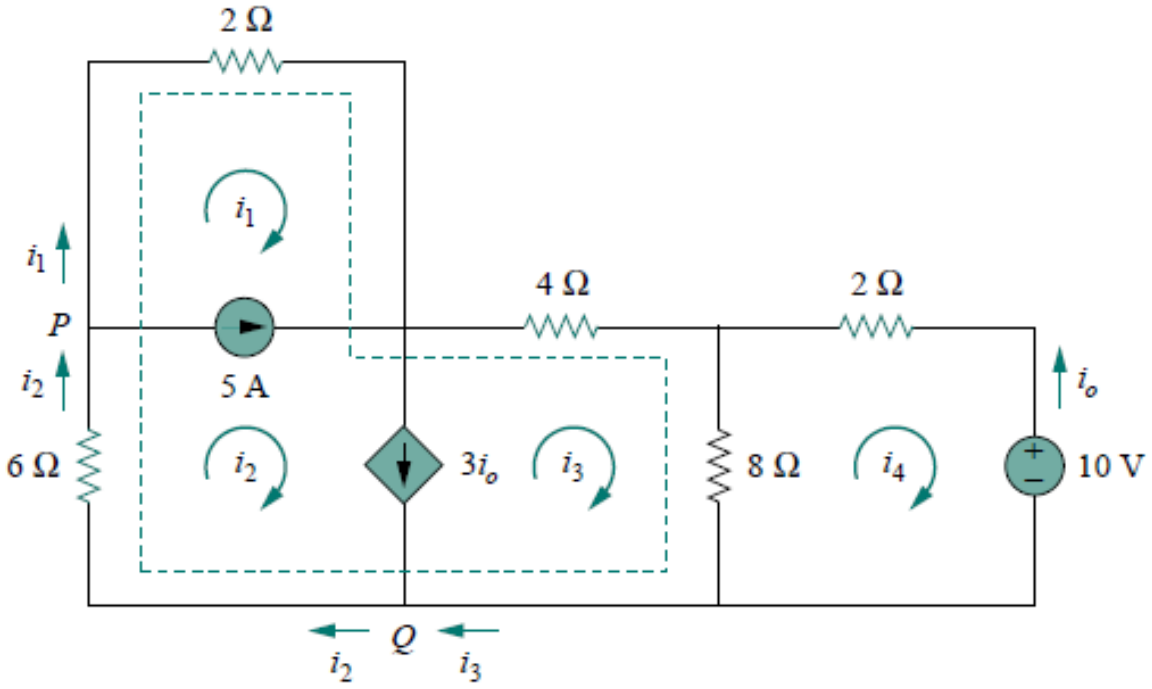
$$i_2 - i_1 = 5A$$

$$i_2 - i_3 = 3i_0 \qquad i_0 = -i_4$$

$$i_2 - i_3 + 3i_4 = 0$$

❑ Modified Mesh Analysis (Modified Mesh-Current Method)

Example: For the circuit given below, find all currents ***<u>using mesh analysis</u>***?



$$2i_1 + 6i_2 + 12i_3 - 8i_4 = 0$$

$$-8i_3 + 10i_4 = -10$$

$$-i_1 + i_2 = 5A$$

$$i_2 - i_3 + 3i_4 = 0$$

4 unknowns and also we have 4 equations ☺

$$i_1 = -7.5A$$

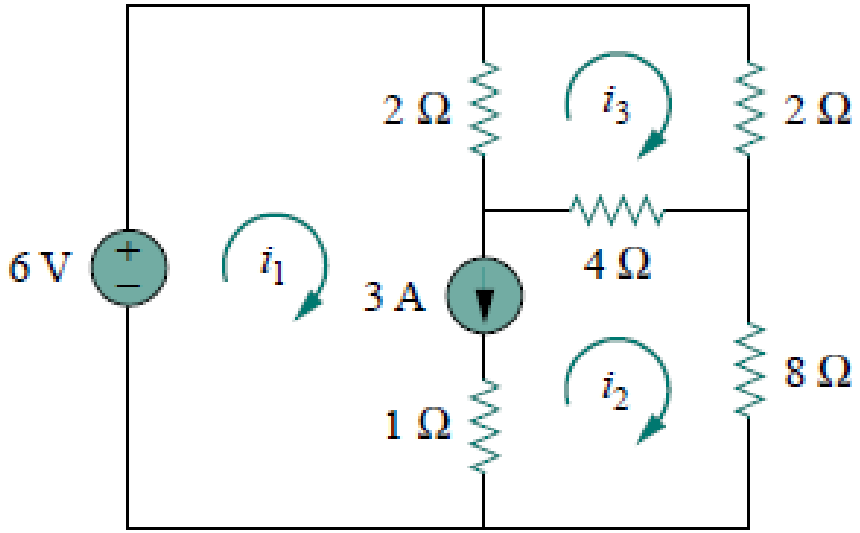
$$i_2 = -2.5A$$

$$i_3 = 3.9236\ A$$

$$i_4 = 2.1429\ A$$

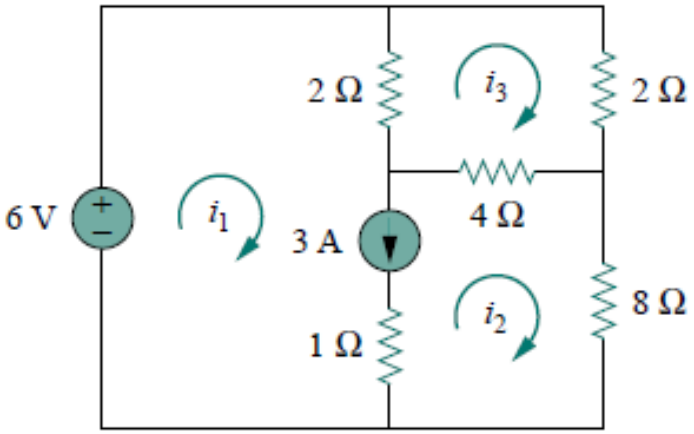
❑ Mesh Analysis (Mesh-Current Method)

Practice Problem : Use mesh analysis to determine i_1, i_2, i_3



❑ Mesh Analysis (Mesh-Current Method)

Practice Problem : Use mesh analysis to determine i_1, i_2, i_3



$Mesh\ 1\ and\ mesh\ 2\ form\ a\ supermesh$!!

$$KVL\ @supermesh \rightarrow 2i_1 + 12i_2 - 6i_3 = 6$$

$$KVL\ @\ mesh3 \rightarrow -2i_1 - 4i_2 + 8i_3 = 0$$

$Constraint\ equation$: $i_1 - i_2 = 3A$

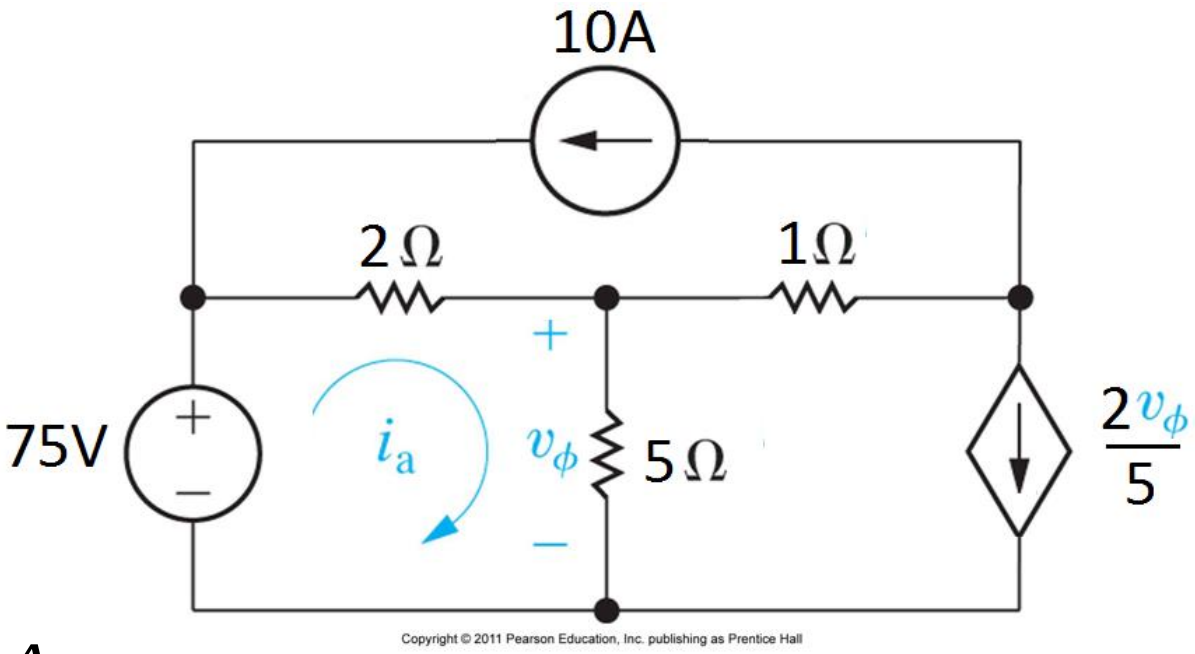
$$i_1 = 3.474\,A$$

$$i_2 = 0.4737\,A$$

$$i_3 = 1.1052\,A$$

❑ Mesh Analysis (Mesh-Current Method)

Practice Problem : Use mesh-current method to find the mesh current i_a in the circuit shown (Assesment problem 4.11 from textbok)

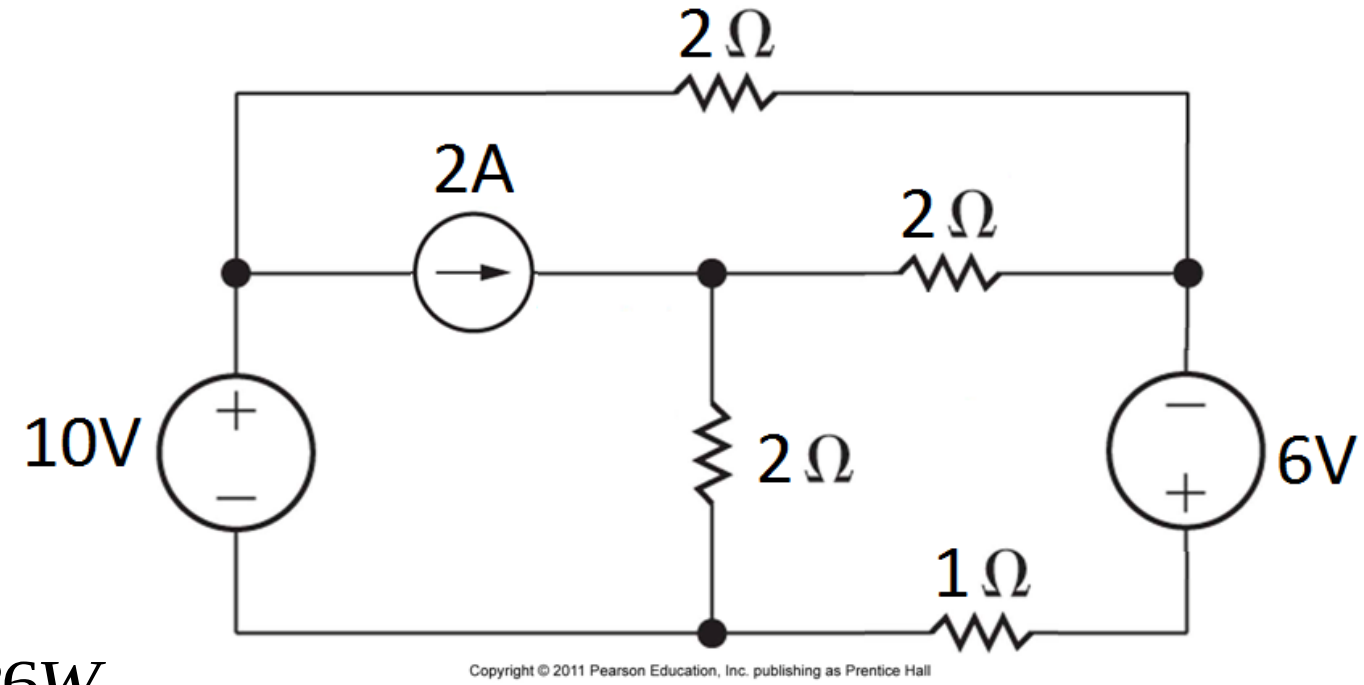


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$$i_a = 15A$$

❑ Mesh Analysis (Mesh-Current Method)

Practice Problem : Use mesh-current method to find the power dissipiated in the 1Ω resistor in the circuit shown (Assesment problem 4.12 from textbok)



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$$P = 36W$$

❑ Mesh and Nodal Analysis

Example 4.6 & 4.7 from textbook are useful to understand when & why are these methods (Mesh and Node Analysis) required.

Try to solve and try to understand these examples.

Also try **assessment problem 4.13** from textbook.

END OF CHAPTER 3, Part 1
(Nodal & Mesh Analysis)

Dr. Yılmaz KALKAN