Highway Design
CE 424

Cenk Ozan, Ph.D.

Adnan Menderes University

Engineering Faculty

Civil Engineering Department

Turning on Curves

-It is desired to travel at a speed on curves as much as on alignments.

-But, small radius curves must be designed especially in mountainous regions .

-Horizontal curve radius depends on vehicle types, vehicle dimensions and speed.

-While radius of curvature is endless at beginning of curve (on alignment), radius of curvature constantly decreases to curve radius on the curve.

Vehicle Stability on Curves

A vehicle which moves from alignment to curve is exposed to centrifugal force. Centrifugal Force has skidding effect and turning over effect.

$$F = \frac{W}{g} \cdot \frac{v^2}{R}$$

W= weight of vehicle (kg)

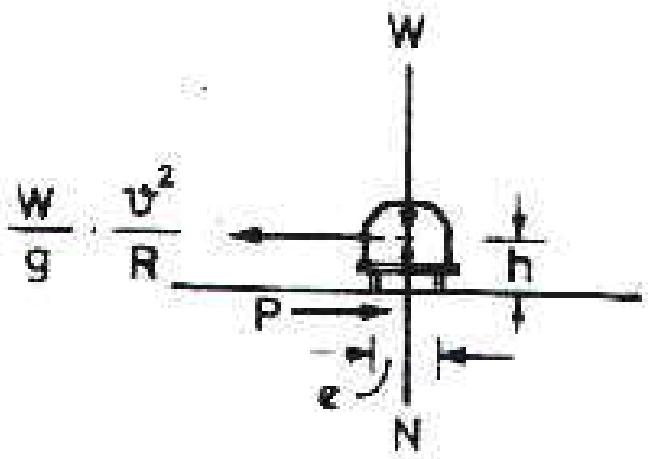
V= speed of vehicle (m/sec)

R= radius of curve (m)

g= acceleration of gravity

F= centrifugal force (kg)

Forces Affecting a Vehicle on Curve



h= height of center of gravity of vehicle from the ground (m)

P= side friction force which balances centrifugal force (kg)

μ_e= coefficient of side friction

$$P = \mu_e . N = \mu_e . W$$

$$\mu_e = \frac{P}{W} = \frac{\frac{W}{g} \cdot \frac{v^2}{R}}{W} = \frac{v^2}{g.R}$$

(it is assumed that P= F)

Vehicle speed km/h, g= 9.81 m/sec^2

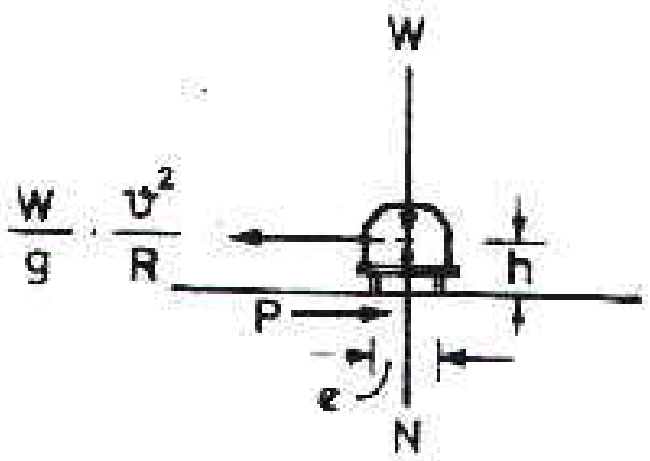
$$\mu_e = \frac{v^2}{127{,}4.R}$$

Skidding speed on the curve without superelevation

$$V_{skid} = 11{,}3\sqrt{\mu_e.R}$$

Moment balances turning over effect of centrifugal force on curves.

Moment = W.(e/2)



$$\mathbf{F.h = W.\frac{e}{2} \quad \Rightarrow \frac{W}{g}.\frac{v^2}{R}.h = W.\frac{e}{2}}$$

$$v = \frac{v}{3{,}6}\, g = 9.81 m/\sec^2$$

$$V_{turnover} = 8{,}0\sqrt{\frac{R.e}{h}}$$

$$\frac{V_{skid}}{V_{turnover}} = \frac{11,3}{8,0}\sqrt{\frac{\mu_e.h}{e}} \quad \frac{V_{skid}}{V_{turnover}} = 1,4\sqrt{\frac{\mu_e.h}{e}}$$

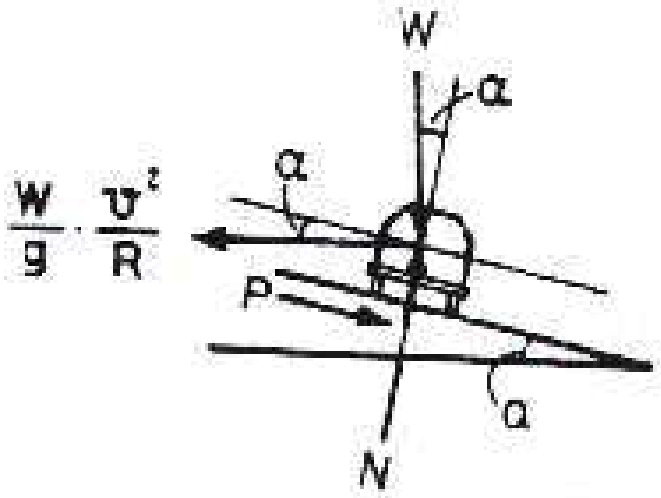
Example: **μ_e= 0,40; h= 0,70m and e= 1,80 m**

$$\frac{V_{skid}}{V_{turnover}} = 1,4\sqrt{\frac{0,40.0,70}{1,80}} = 0,56$$

Accidents on the curves occurs to skid rather than turning over.

In practice, superelevation is designed to balance skidding and turning over effects of centrifugal force which affects negatively vehicle's stability.

Critical speeds cause skidding and turning over on superelevation with curves:



$$V_{skid} = 11{,}3\sqrt{\frac{R(\mu_e + tg\alpha)}{1 - \mu_e . tg\alpha}}$$

İf Tgα= 0

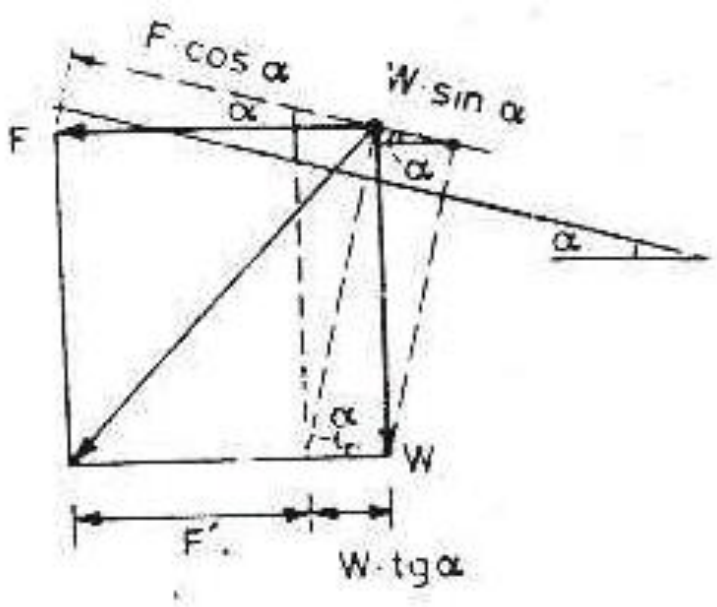
$$V_{skid} = 11.3 * \sqrt{\mu_e * R}$$

$$V_{turnover} = 11{,}3\sqrt{\frac{R(h.tg\alpha + \frac{e}{2})}{h - tg\alpha.\frac{e}{2}}}$$

Minimum radius of curves for design speed:

$$R_{\min(skid)} \geq \frac{V_p{}^2.(1 - \mu_e.tg\alpha)}{127{,}4.(\mu_e + tg\alpha)} \qquad R_{\min(turnover)} \geq \frac{V_p{}^2.(h - \frac{e}{2}.tg\alpha)}{127{,}4.(h.tg\alpha + \frac{e}{2})}$$

Lateral Acceleration and Rate of Change of Lateral Acceleration



$$F' = F - W.tg\alpha$$

$$m.p = m.\frac{v^2}{R} - m.g.tg\alpha$$

$$d = tg\alpha = d/100$$

p= lateral acceleration (m/sec^2)

V= vehicle speed (m/sec)

m= mass of vehicle (kg)

d= superelevation (%)

R= radius of curve (m)

$$p = \frac{v^2}{R} - g.\frac{d}{100}$$

$$p = \frac{v^2}{12{,}96.R} - 0{,}0981.d$$

Lateral acceleration occuring on curve the change in unit time is called to Rate of Change of Lateral Acceleration (Sademe).

t= L/v or 3,6L/v

$$p' = \frac{dp}{dt} = \frac{p}{t}$$

$$p' = \frac{v^2}{12{,}96.R} \cdot \frac{v}{3{,}6.L} - 0{,}0981.d.\frac{v}{3{,}6.L}$$

$$p' = \frac{v^3}{46{,}7.R.L} - \frac{v.d}{36{,}7.L}$$

Rate of Change of Lateral Acceleration is felt from P'= 0,3 m/sec³.

Max. Rate of Change of Lateral Acceleration is 0,6 m/sec³.

Minimum Radius of Curve

$$\frac{v^2}{g.R} = tg\alpha + \mu_e + \mu_e . \frac{v^2}{g.R} . tg\alpha$$

$$\frac{v^2}{g.R} = tg\alpha + \mu_e$$

$$\frac{v^2}{127.R} = d + \mu_e$$

$$R_{min} = \frac{v_p^{\ 2}}{127(d + \mu_e)}$$

As can be seen in next formula, Centrifugal force is balanced superelevation and side friction

Side friction coefficent on dry ways: 0,40-0,50.

If speed increases, it decreases.

Exchange rate of side friction coefficient

Design Speed (km/h)	**50**	**70**	**90**	**100**	**110**	**120**
Side Friction Coefficient	**0,16**	**0,15**	**0,13**	**0,13**	**0,12**	**0,12**

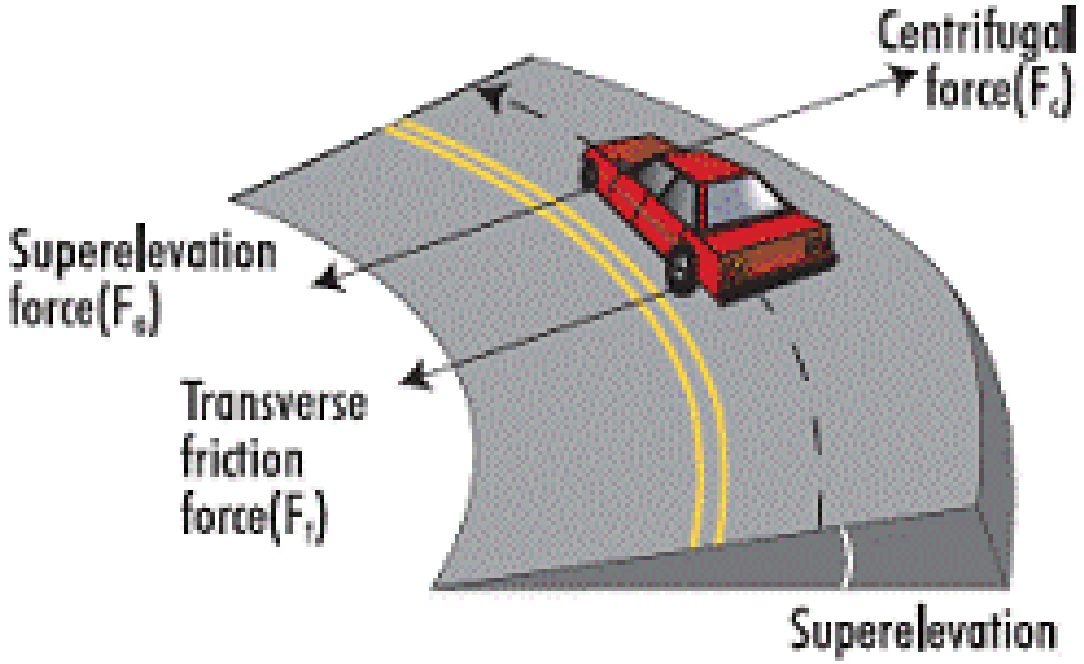
Superelevation

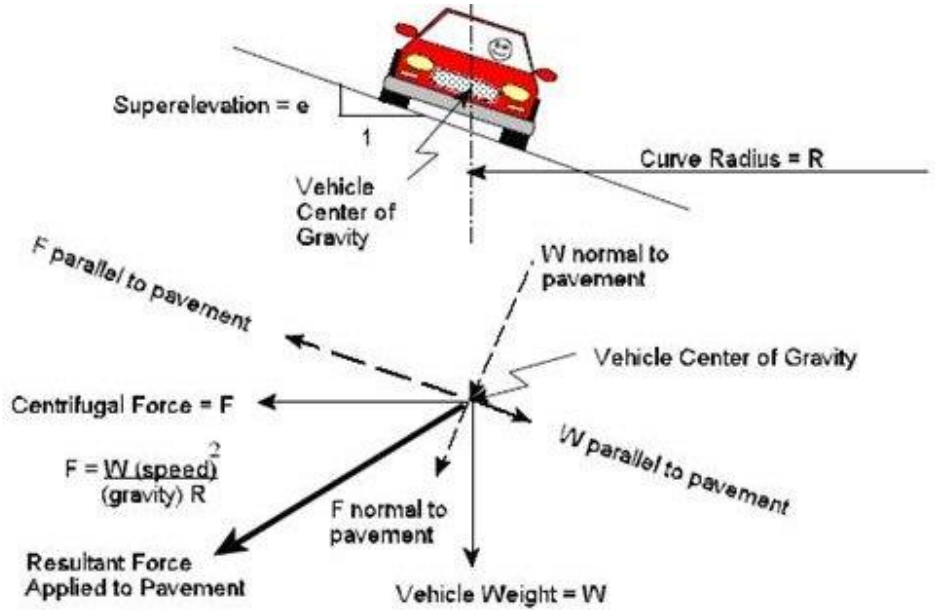
If lateral acceleration is balanced with superelevation, lateral acceleration means zero. It is called theoretical superelevation.

It is obtained that lateral acceleration formula is equalized to zero.

$$p = \frac{v^2}{12{,}96.R} - 9{,}81.d = 0 \; and \; v = v_p$$

$$\text{superelevation}_{teo} = 0{,}00786.\frac{v_p{}^2}{R}$$


Centrifugal force(F_c)
Superelevation force(F_e)
Transverse friction force(F_t)
Superelevation

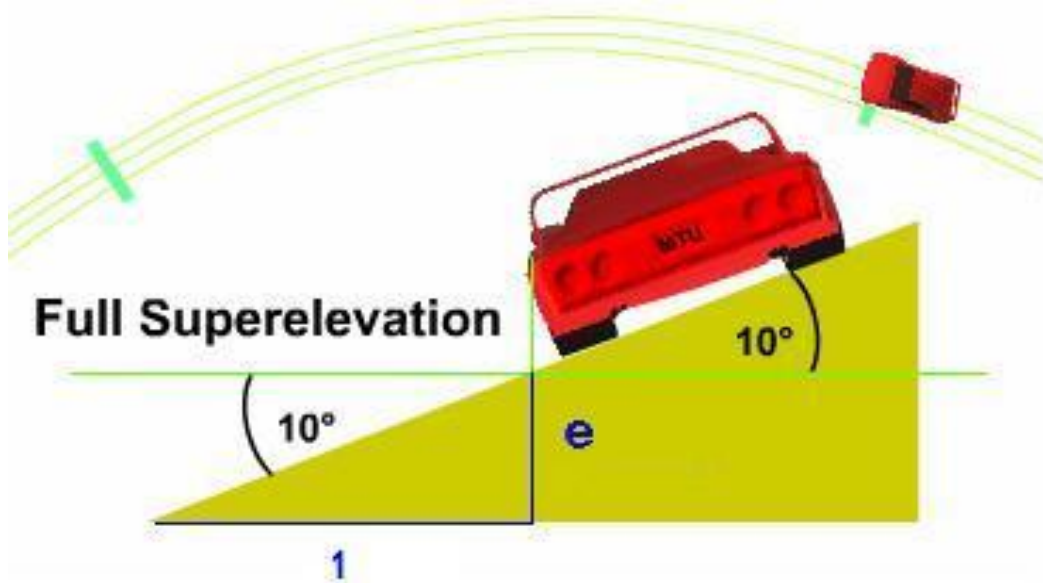

Superelevation = e
1
Curve Radius = R
Vehicle Center of Gravity
F parallel to pavement
W normal to pavement
Vehicle Center of Gravity
Centrifugal Force = F
F = W (speed)^2 / (gravity) R
W parallel to pavement
F normal to pavement
Resultant Force Applied to Pavement
Vehicle Weight = W

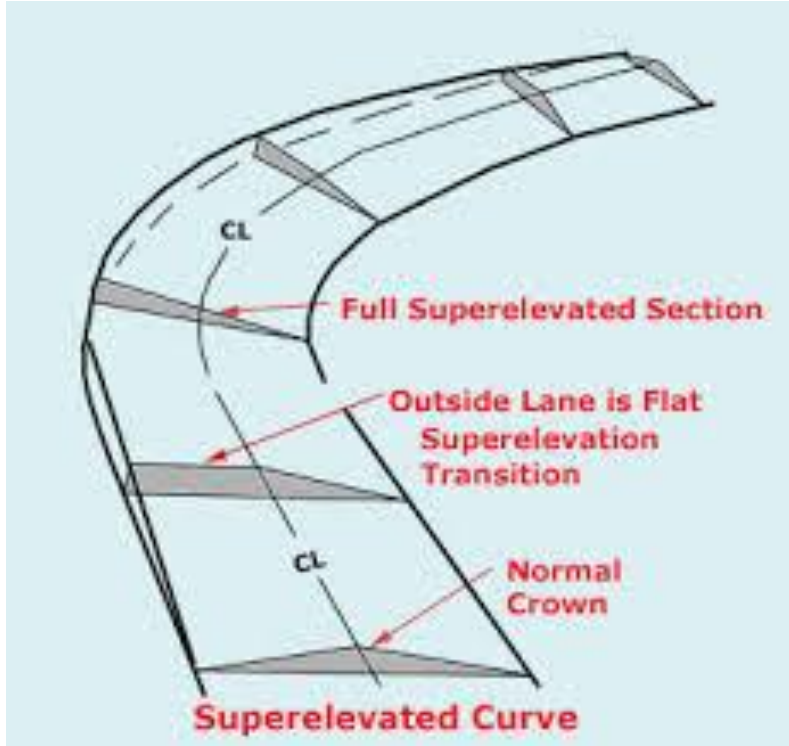
Disadvantages of Excessive Superelevation

-If vehicles stop on the curve or travel slowly, vehicles can skid or turn over to center of curves.

-Thus, Max. Superelevation is %8-10.

-It is kept lower in snowy, frozen regions and urban highways


Full Superelevation
10°
10°
e
1


CL
Full Superelevated Section
Outside Lane is Flat
Superelevation
Transition
CL
Normal
Crown
Superelevated Curve


superelevation

In fact, lateral acceleration is balanced with superelevation and side friction. In this case, superelevation;

$$\text{superelevation} = 0{,}00393.\frac{v_p^2}{R}$$

Superelevation Formula in Turkey

$$\text{superelevation} = 0{,}00443.\frac{v_p^2}{R}$$

Max. superelevation: %10

V_p= design speed (km/h)

R= radius of curve (m)

$$L_R = 0{,}0354 * \frac{v_p^3}{R}$$

L_R= length of Raccordement

$L_{R(min)}$= 45 m

Superelevation Applications

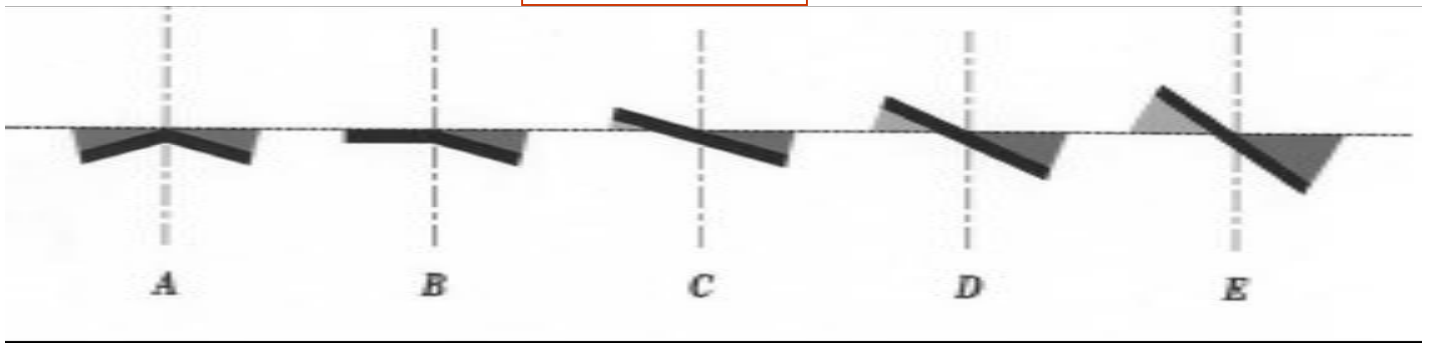
There are three basic methods for developing superelevation on a crowned pavement leading into and coming out of a horizontal curve.

In the most commonly used method, case I, the pavement edges are revolved about the centerline. Thus, the inner edge of the pavement is depressed by half of the superelevation and the outer edge raised by the same amount.

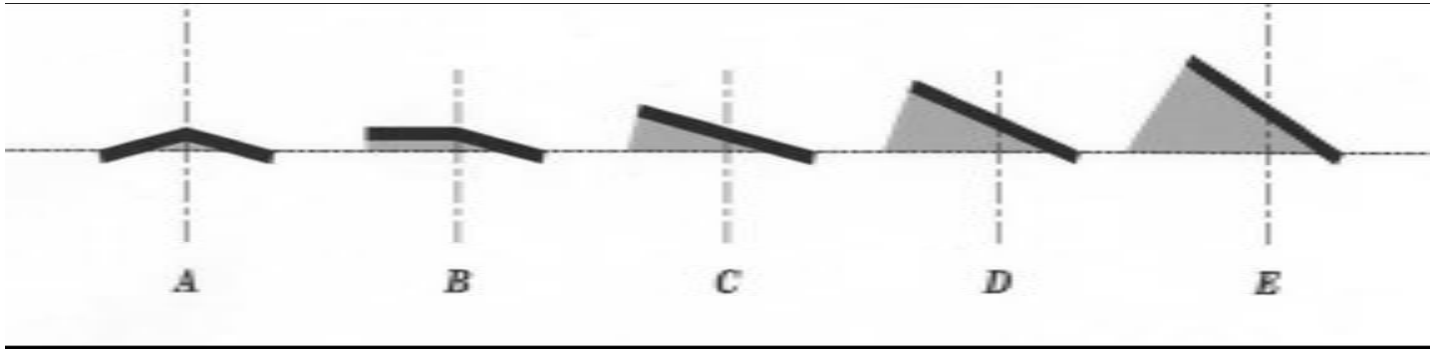
Case II shows the pavement revolved about the inner or lower edge of pavement, and case III shows the pavement revolved about the outer or higher edge of pavement.

Case II can be used where off-road drainage is a problem and lowering the inner pavement edge cannot be accommodated. The superelevation on divided roadways is achieved by revolving the pavements about the median pavement edge. In this way, the outside (high side) roadway uses case II, while the inside (low side) roadway uses case III. This helps control the amount of “distortion” in grading the median area.

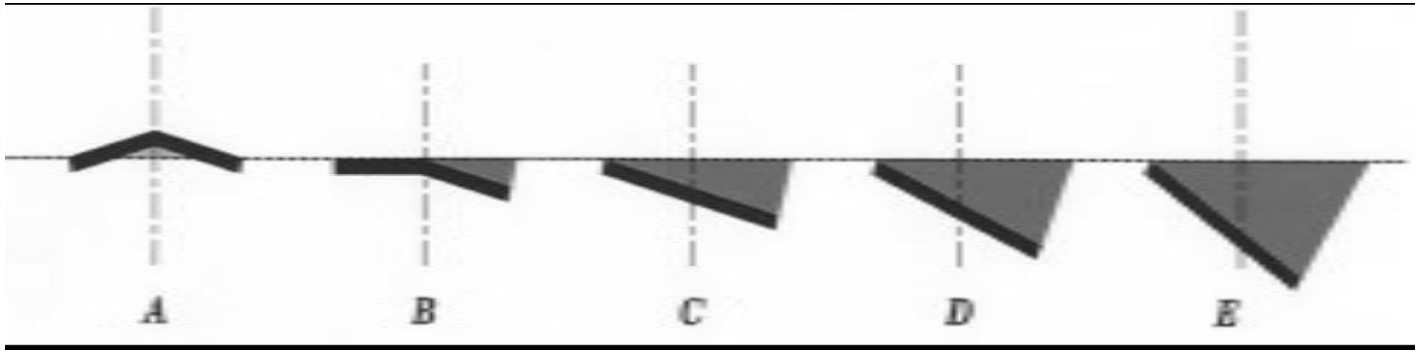
Case I

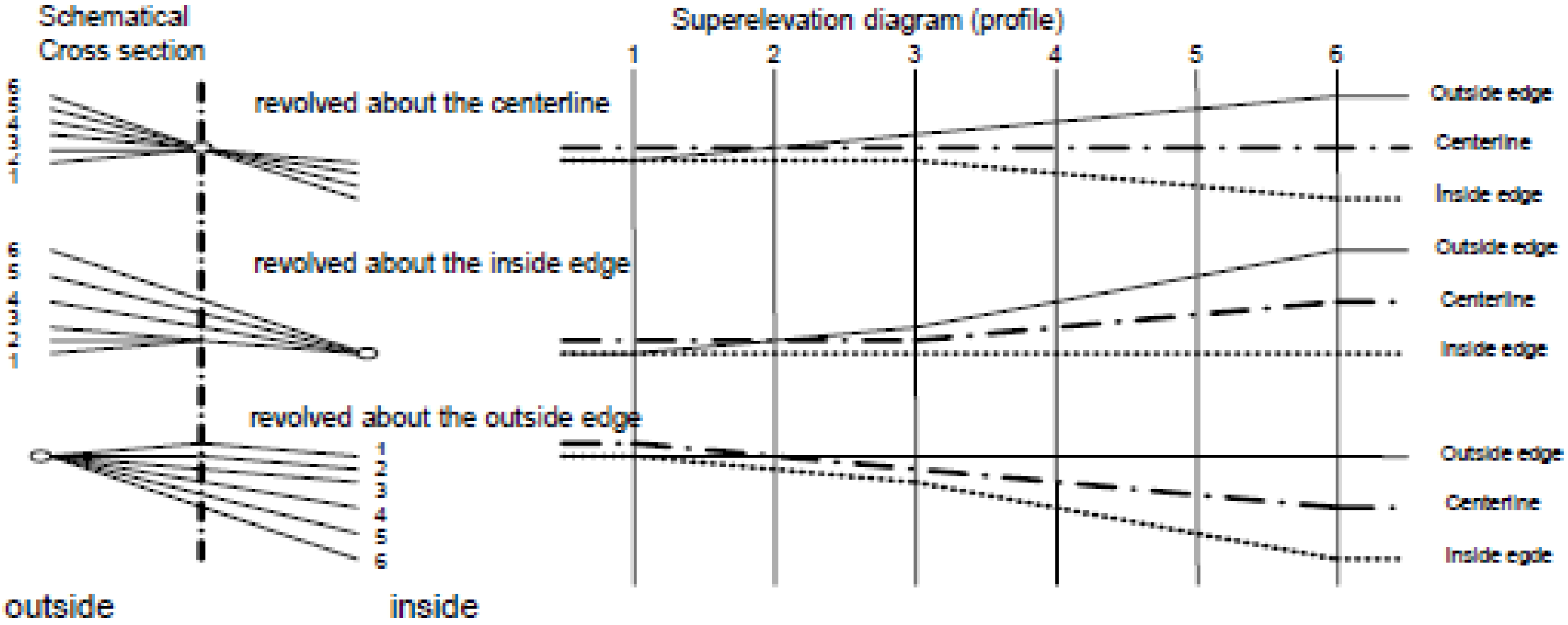


Case II

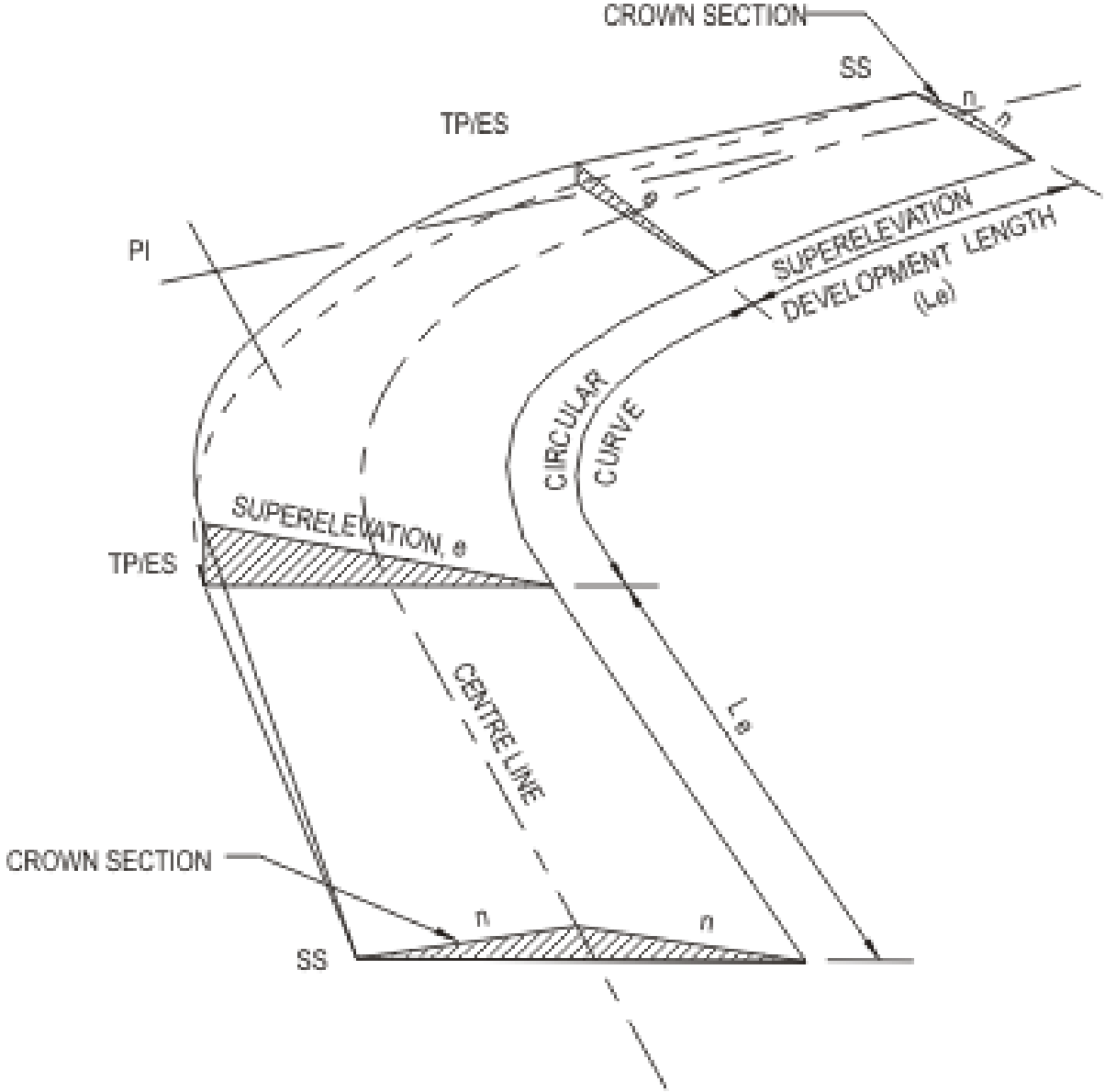


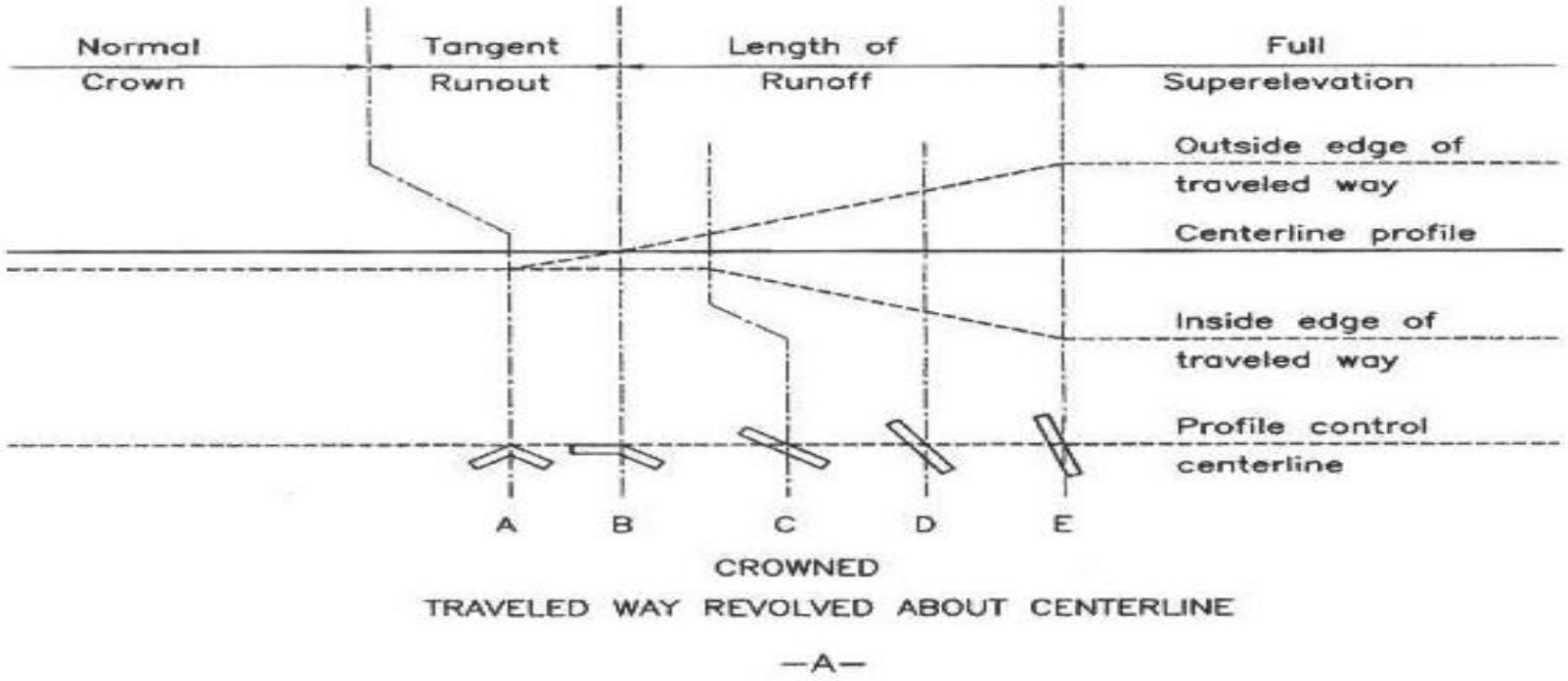
Case III



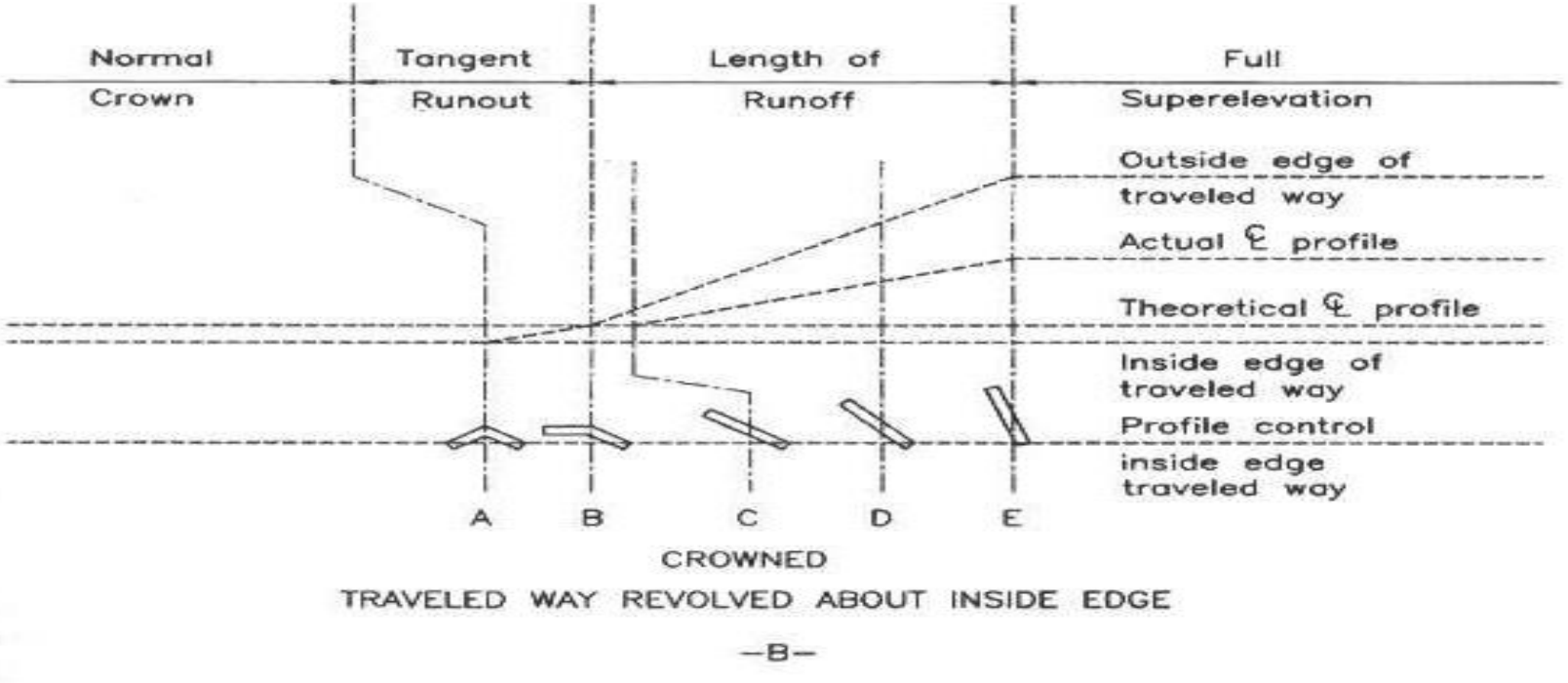


Superelevation Applications

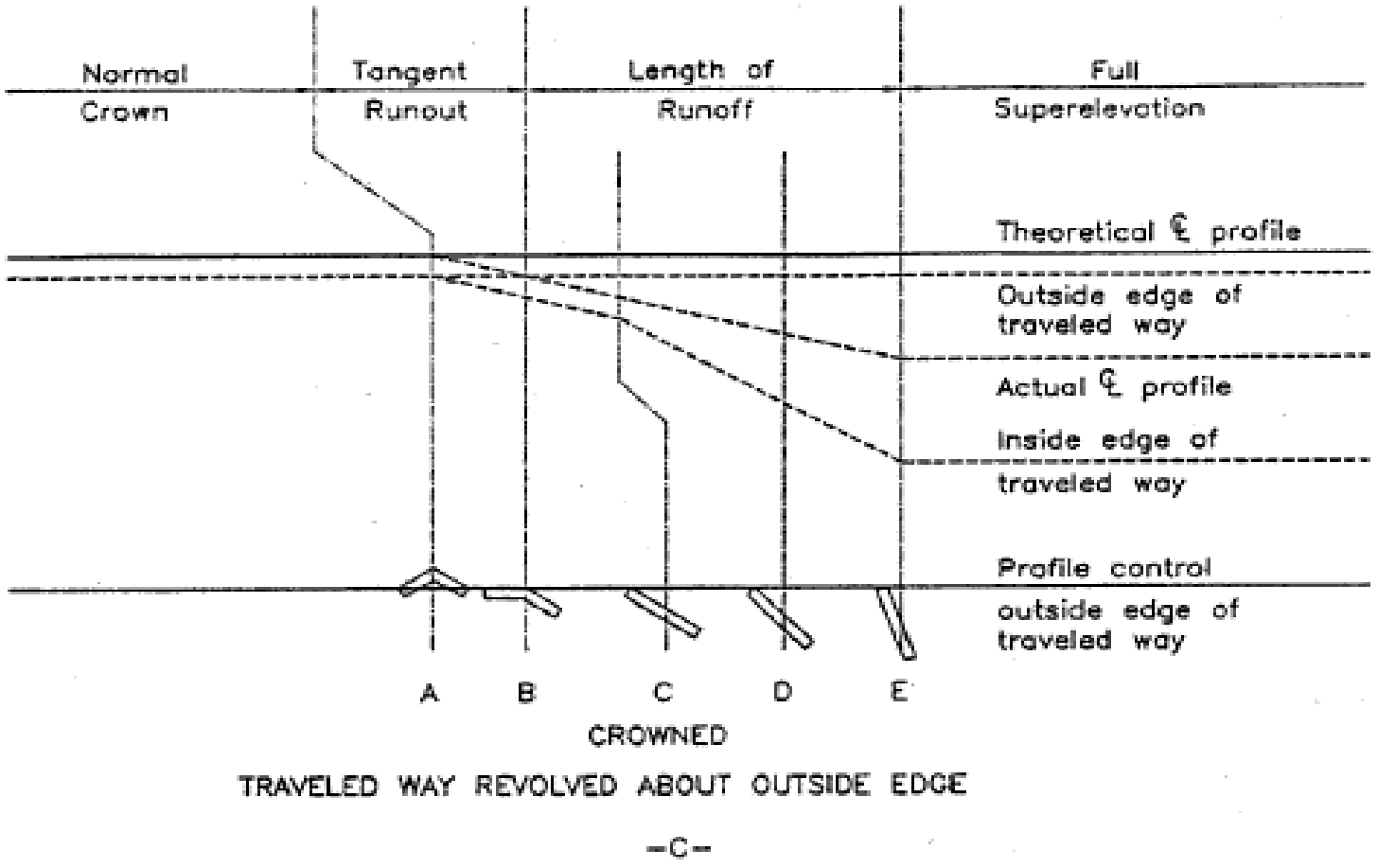

CROWN SECTION
SS
n
2
TP/ES
e
PI
SUPERELEVATION
DEVELOPMENT LENGTH
(Le)
CIRCULAR
CURVE
SUPERELEVATION, e
TP/ES
CENTRE LINE
Le
CROWN SECTION
n
n
SS



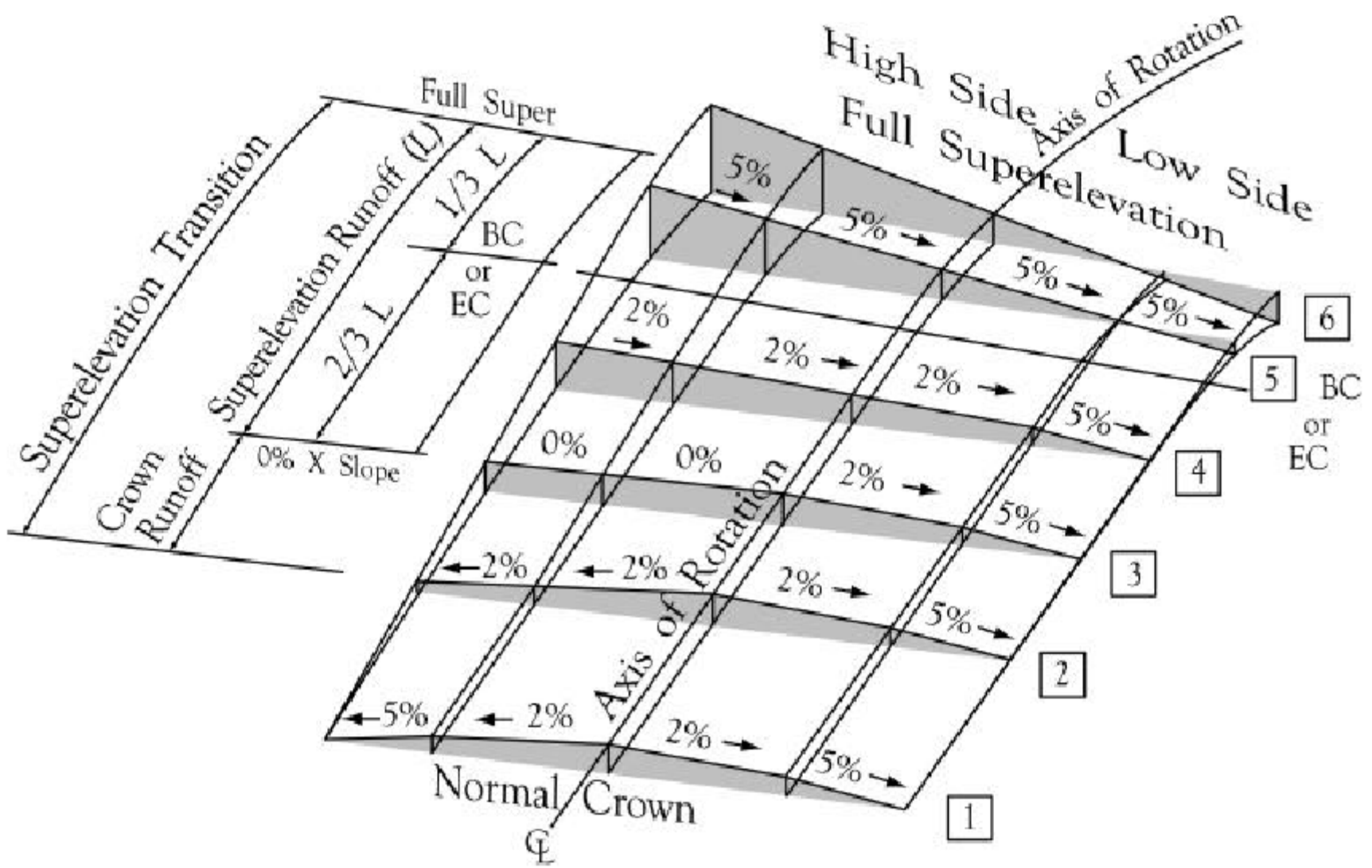
Superelevation Applications



Superelevation Applications

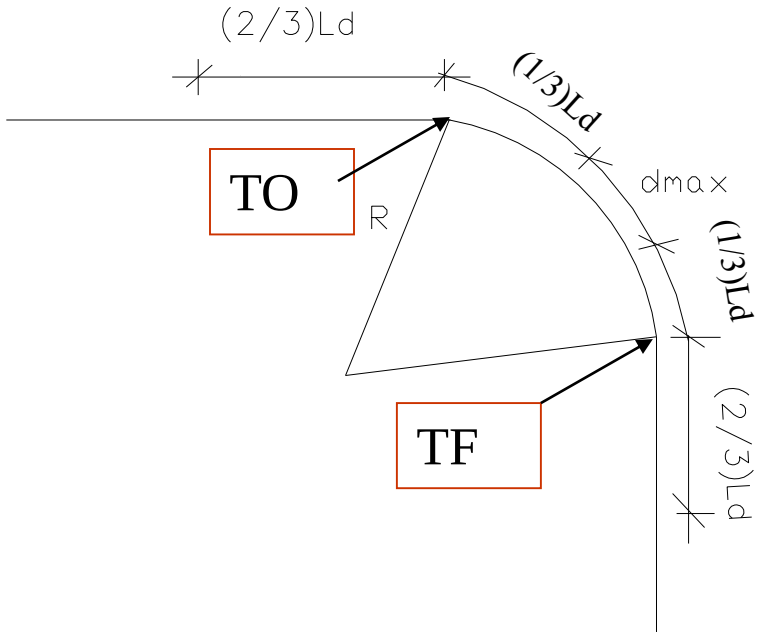


Superelevation Applications



Elements of a Superelevation Transition (Right Curve)

Superelevation Application Without Transition Curve



Transition Curves

Vehicles which move from alignment to curve are affected by centrifugal force. That centrifugal force is balanced with superelevation and transition curves which are placed between alignment and curve.

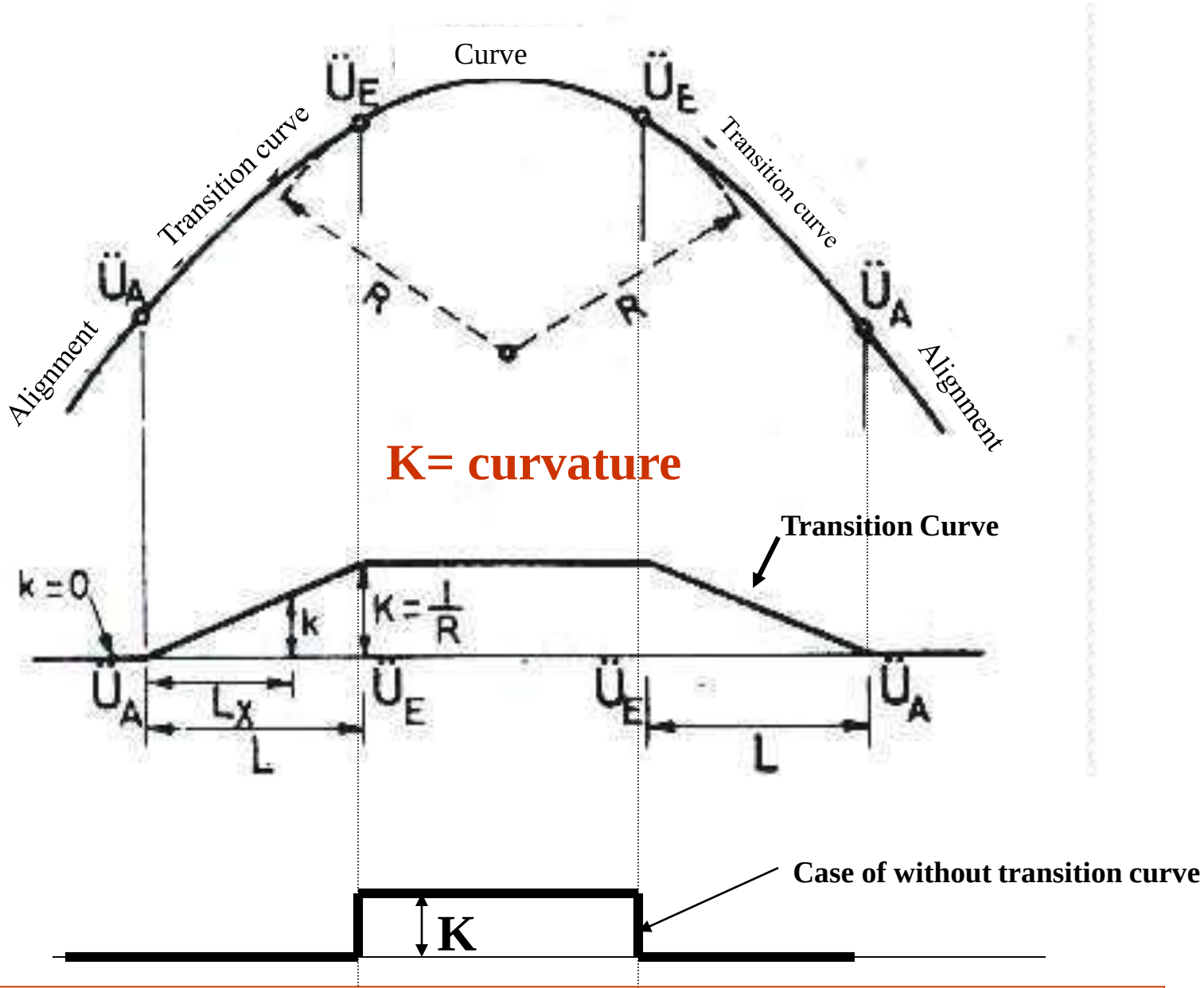
Due to transition curve, centrifugal force effect is distributed equally along transition curve and removed at beginning of curve.

Conditions about Transition Curves

-Vehicle can be travelled on transition curve with a speed as much as on alignment.

- When vehicle steering is turned constant angular speed, vehicle should reach the largest rotation angle at the entrance of curve.

- Transition curve should be tangent to alignment at the beginning (radius of curvature is endless) and should be tangent to circular curve at the end (radius of curvature is equal to radius of curve).



Curvature change on horizontal curves

L= length of transition curve

$\ddot{U}_A$= start of transition curve

$\ddot{U}_E$= end of transition curve

k= curvature

-Curvature at L_x distance

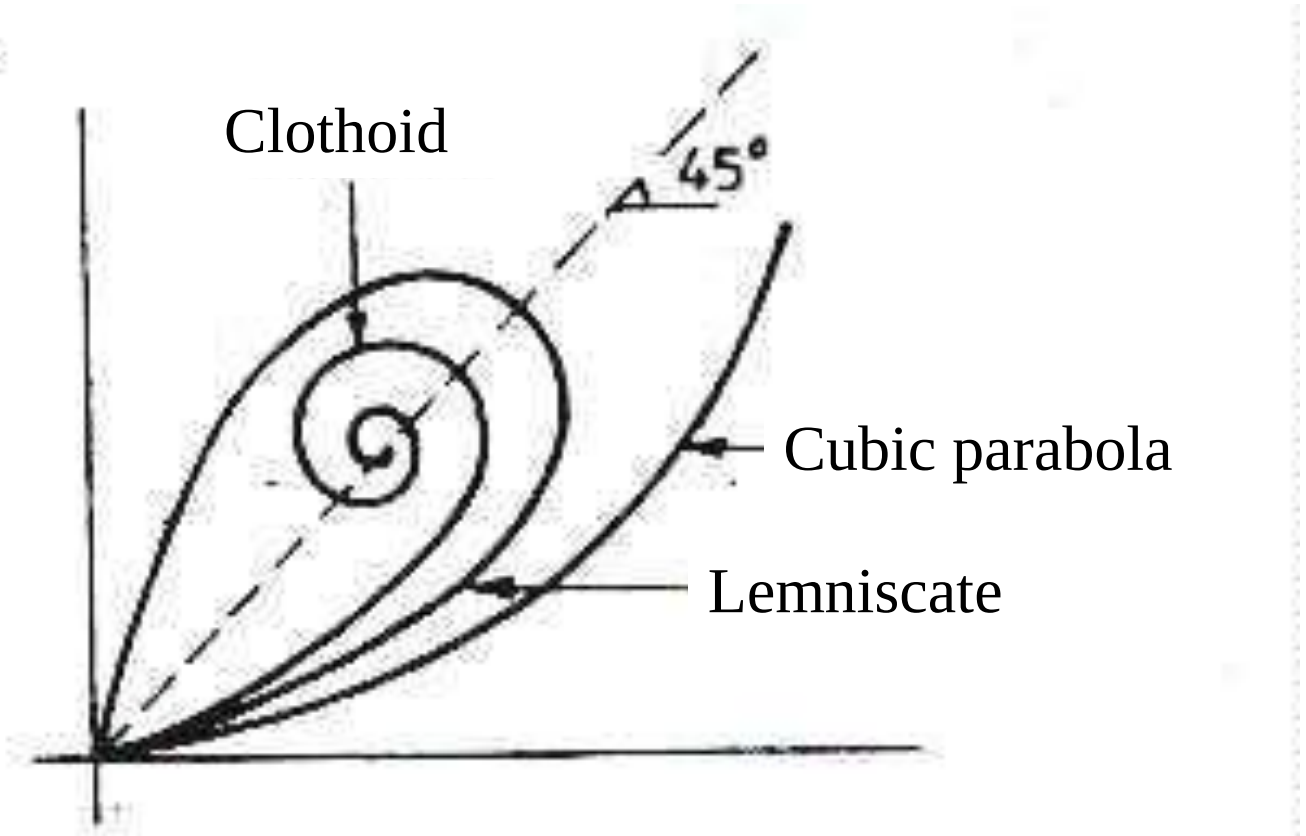
$k=(K/L)*L_x$

-Curvature at the point of $\ddot{U}_E$ ➔ K=1/r

$k= L_x/(L*R)$

Transition Curve Types

- Clothoid (the most widely used in the world)
- Lemniscate
- Cubic Parabola



Transition curves

Length of Transition Curve

It is desired that vehicles should not exceed certain a value of Sademe on the account of length of transition curve.

Vehicle has zero lateral acceleration at the beginning of transition curve. When entering curve, vehicle reachs v^2/R value of lateral acceleration. This changing occurs during t= L/v.

Lateral acceleration occuring on curve the change in unit time is called to Rate of Change of Lateral Acceleration (sademe).

$$p' = \frac{\frac{v^2}{R}}{\frac{L}{v}} = \frac{v^3}{R.L}$$

Speed km/h

$$L = \frac{V_p^{\,3}}{46{,}7.R.p'}$$

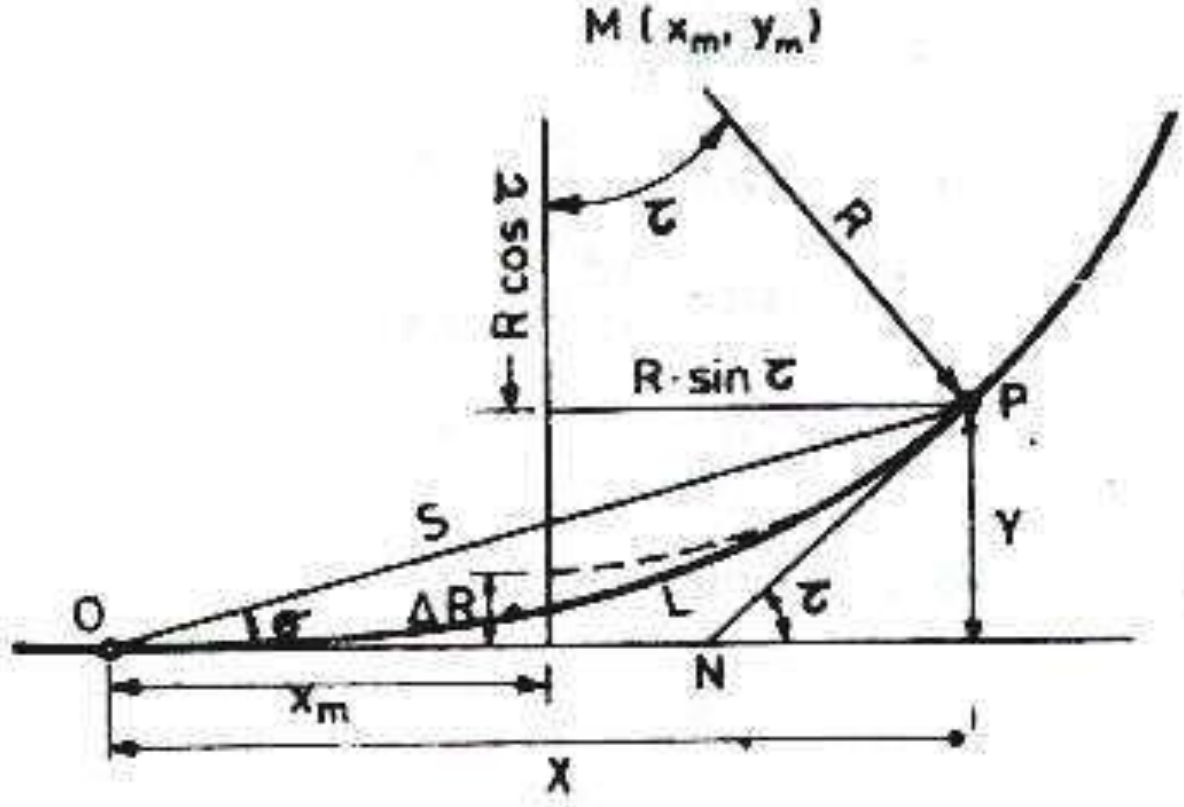
Rate of Change of Lateral Acceleration (sademe)=p'= 0,3-0,6 m/sec^3

Clothoid

It is used in high-standard roads

R.L= A^2 (A= clothoid parameter)

Clothoid is actually a spiral. The spiral is the one most commonly used in highway design. The degree of curve varies gradually from zero at the tangent end to the degree of the circular arc at the curve end.
By definition, the degree of curve at any point along the spiral varies directly with the length measured along the spiral. In the case where a spiral transition connects two simple curves, the degree of curve varies directly from that of the first circular arc to that of the second circular arc.


M (x_m, y_m)
τ
R
R cos τ
R · sin τ
P
Y
S
ΔR
L
τ
O
ε
N
x_m
X

O= the beginning point of the clothoid

P= the end point of the clothoid

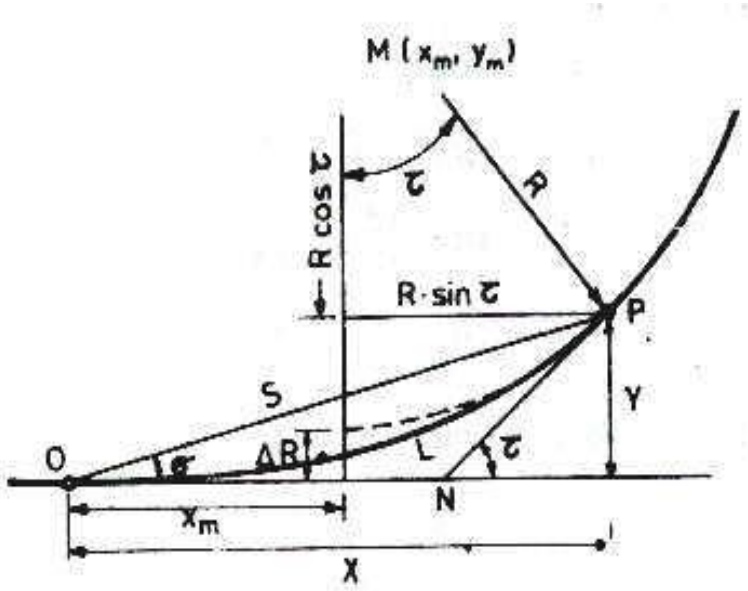
X,Y= coordinates of point of P

X_m, y_m= coordinates of center point of curvature

NP= T_k= length of short tangent

NO= T_u= length of long tangent

ΔR= transition proportion



A point to be taken into account in clothoid tangent with the horizontal axis is τ that point, the value of angle radians

$$\tau = \frac{L}{2R}$$

$$\mathbf{X} = \mathbf{L} - \frac{\mathbf{L}^5}{40.\mathbf{A}^4} + \frac{\mathbf{L}^9}{3456.\mathbf{A}^8} - \ldots$$

$$\mathbf{Y} = \frac{\mathbf{L}^3}{6.\mathbf{A}^2} - \frac{\mathbf{L}^7}{336.\mathbf{A}^6} + \frac{\mathbf{L}^{11}}{42240.\mathbf{A}^{10}} - \ldots$$

$$\mathbf{x_m} = \mathbf{X} - \mathbf{R}.\sin\boldsymbol{\tau}$$

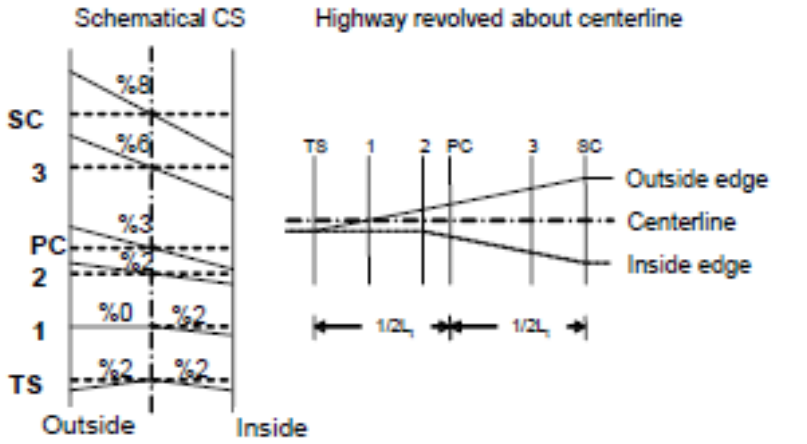
$$\mathbf{y_m} = \mathbf{Y} + \mathbf{R}.\cos\boldsymbol{\tau}$$

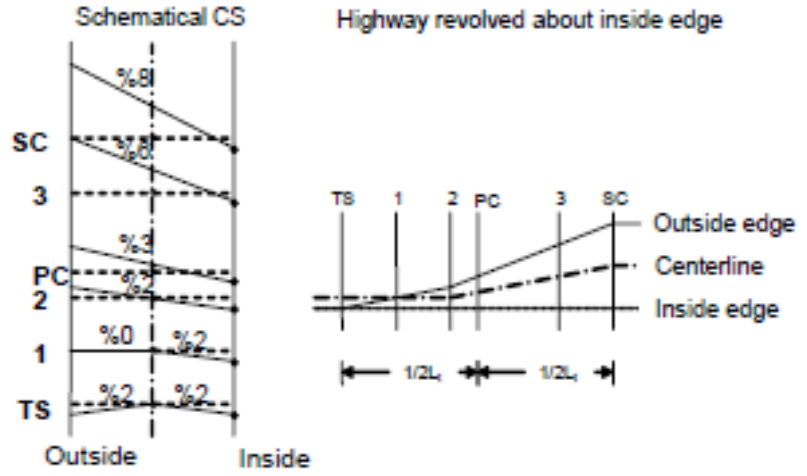
$$\mathbf{T_k} = \frac{\mathbf{y}}{\sin\boldsymbol{\tau}} \qquad \mathbf{T_u} = \mathbf{X} - \mathbf{Y}.\cot\mathbf{g}\boldsymbol{\tau}$$

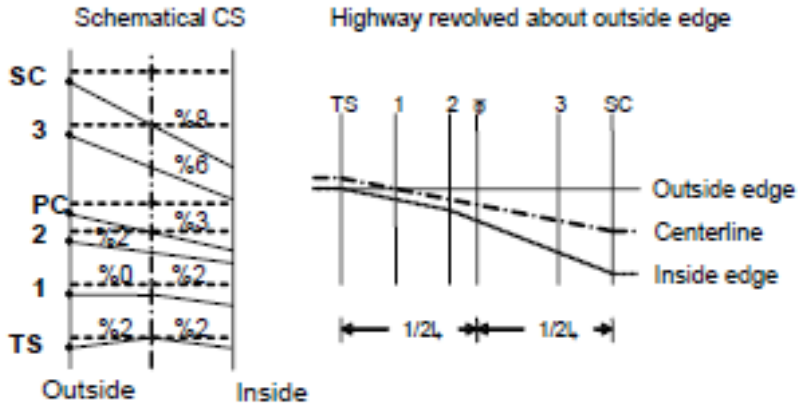
$$\Delta\mathbf{R} = \mathbf{y_m} - \mathbf{R} = \mathbf{Y} - \mathbf{R}(1 - \cos\boldsymbol{\tau})$$

$$\mathbf{S} = \sqrt{\mathbf{x}^2} + \mathbf{y}^2$$

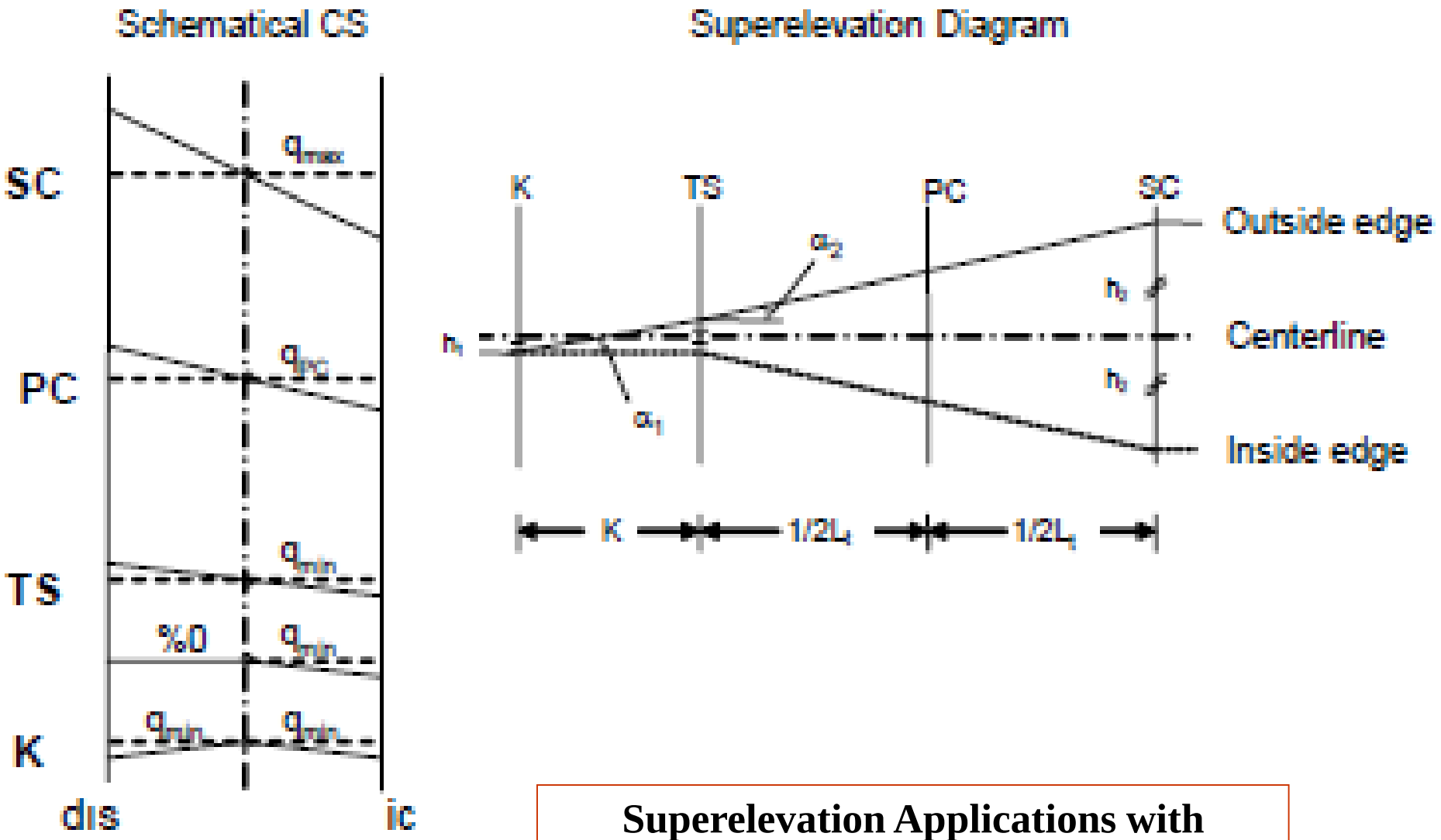
$$\mathbf{tg}\boldsymbol{\sigma} = \frac{\mathbf{Y}}{\mathbf{X}} \qquad \boldsymbol{\sigma} = \mathbf{arctg}\frac{\mathbf{y}}{\mathbf{x}}$$



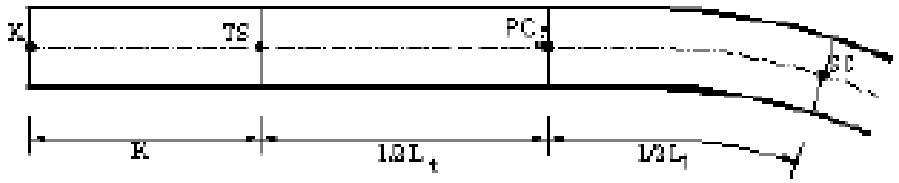


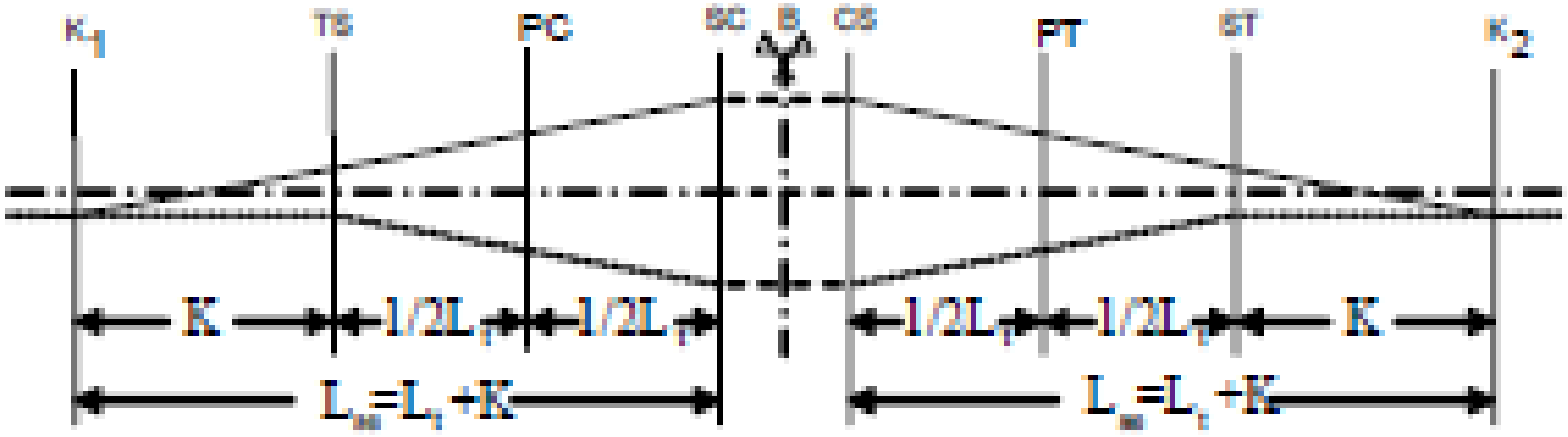


Superelevation Applications with transition curve without runout

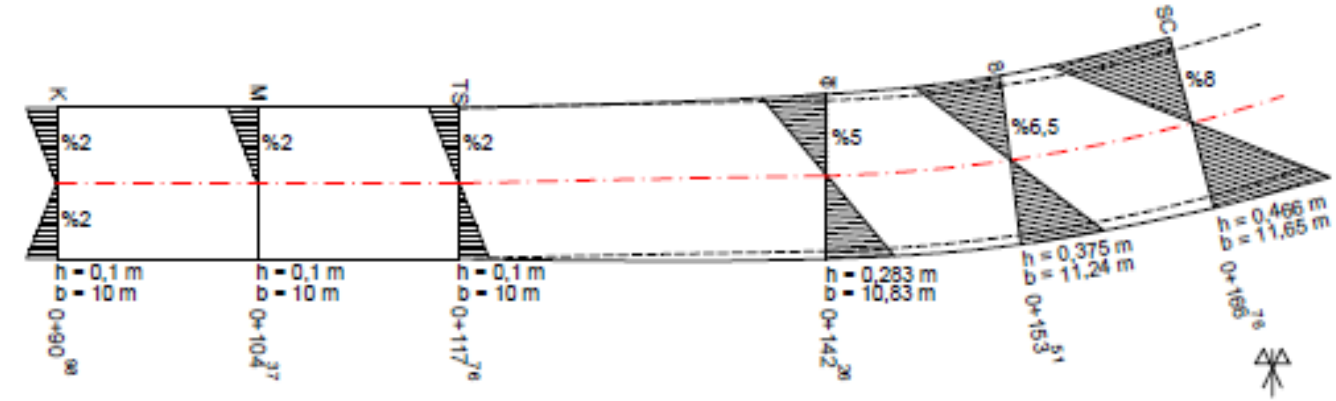


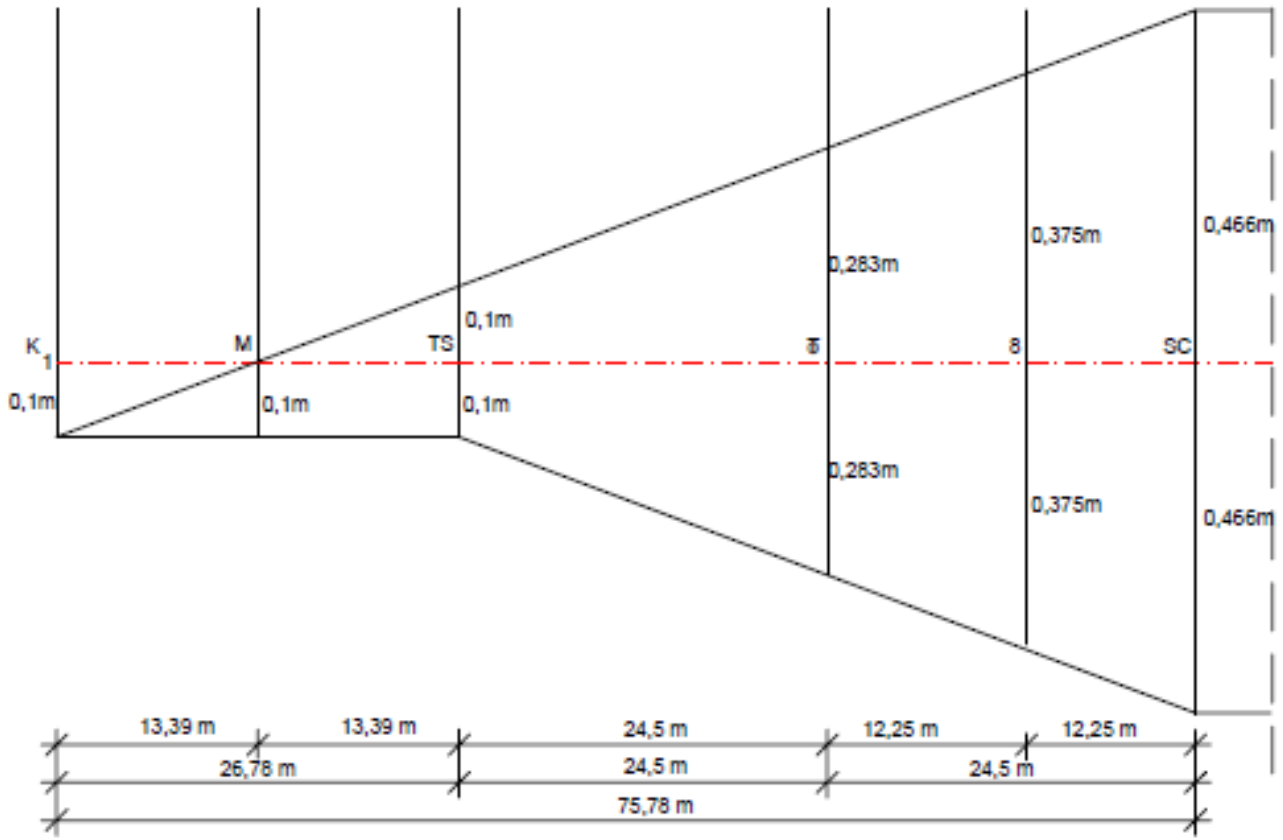
Superelevation Applications with transition curve (revolved about the centerline with runout

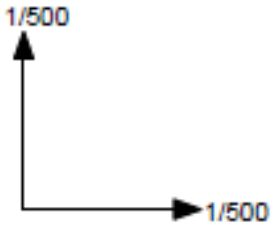

K
TS
PC
K
1/2L
1/2L

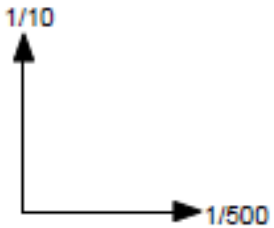

K1
TS
PC
SC
B
CS
PT
ST
K2
K
1/2L
1/2L
1/2L
1/2L
K
L=L +K
L=L +K

Curve No: 1		Inside (Left) Edge			Outside (Right) Edge		
Station	Km	q (%)	h(m)	Grade Elevation (m)	q (%)	h(m)	Grade Elevation (m)
K_1	$0+393^{55}$	-2,00	-0,105	938,47	-2,00	-0,105	938,47
H_4	$0+400^{00}$	-2,00	-0,105	938,29	-1,81	-0,095	938,30
9	$0+422^{22}$	-2,00	-0,105	937,64	-1,16	-0,061	937,68
10	$0+466^{86}$	-2,00	-0,105	936,35	+0,15	+0,008	936,46
H_5	$0+500^{00}$	-2,00	-0,105	935,39	+1,12	+0,059	935,55
11	$0+527^{12}$	-2,00	-0,105	934,60	+1,91	+0,100	934,80
TS	$0+530^{14}$	-2,00	-0,105	934,51	+2,00	+0,105	934,72
12	$0+548^{61}$	-2,54	-0,133	933,94	+2,54	+0,133	934,20
TO	$0+570^{14}$	-3,17	-0,166	933,28	+3,17	+0,166	933,62
H_6	$0+600^{00}$	-4,04	-0,212	932,37	+4,04	+0,212	932,79
SC	$0+610^{14}$	-4,34	-0,228	932,06	+4,34	+0,228	932,52
13	$0+650^{00}$	-4,34	-0,228	930,90	+4,34	+0,228	931,36
H_7	$0+700^{00}$	-4,34	-0,228	929,45	+4,34	+0,228	929,91
14	$0+750^{30}$	-4,34	-0,228	927,99	+4,34	+0,228	928,45
B	$0+769^{04}$	-4,34	-0,228	927,44	+4,34	+0,228	927,90
H_8	$0+800^{00}$	-4,34	-0,228	926,55	+4,34	+0,228	927,01
15	$0+819^{09}$	-4,34	-0,228	925,99	+4,34	+0,228	926,45
16	$0+840^{14}$	-4,34	-0,228	925,38	+4,34	+0,228	925,84
17	$0+873^{94}$	-4,34	-0,228	924,36	+4,34	+0,228	924,82
18	$0+887^{44}$	-4,34	-0,228	923,94	+4,34	+0,228	924,40
H_9	$0+900^{00}$	-4,34	-0,228	923,54	+4,34	+0,228	924,00
CS	$0+927^{94}$	-4,34	-0,228	922,64	+4,34	+0,228	923,10
19	$0+938^{60}$	-4,03	-0,212	922,31	+4,03	+0,212	922,73
20	$0+953^{10}$	-3,60	-0,189	921,84	+3,60	+0,189	922,22
TF	$0+967^{94}$	-3,17	-0,166	921,35	+3,17	+0,166	921,69
H_{10}	$1+000^{00}$	-2,23	-0,117	920,27	+2,23	+0,117	920,51
ST	$1+007^{94}$	-2,00	-0,105	920,00	+2,00	+0,105	920,21
21	$1+018^{76}$	-2,00	-0,105	919,62	+1,68	+0,088	919,81
22	$1+045^{48}$	-2,00	-0,105	918,62	+0,90	+0,047	918,77
H_{11}	$1+100^{00}$	-2,00	-0,105	916,52	-0,70	-0,037	916,58
K_2	$1+144^{53}$	-2,00	-0,105	914,70	-2,00	-0,105	914,70







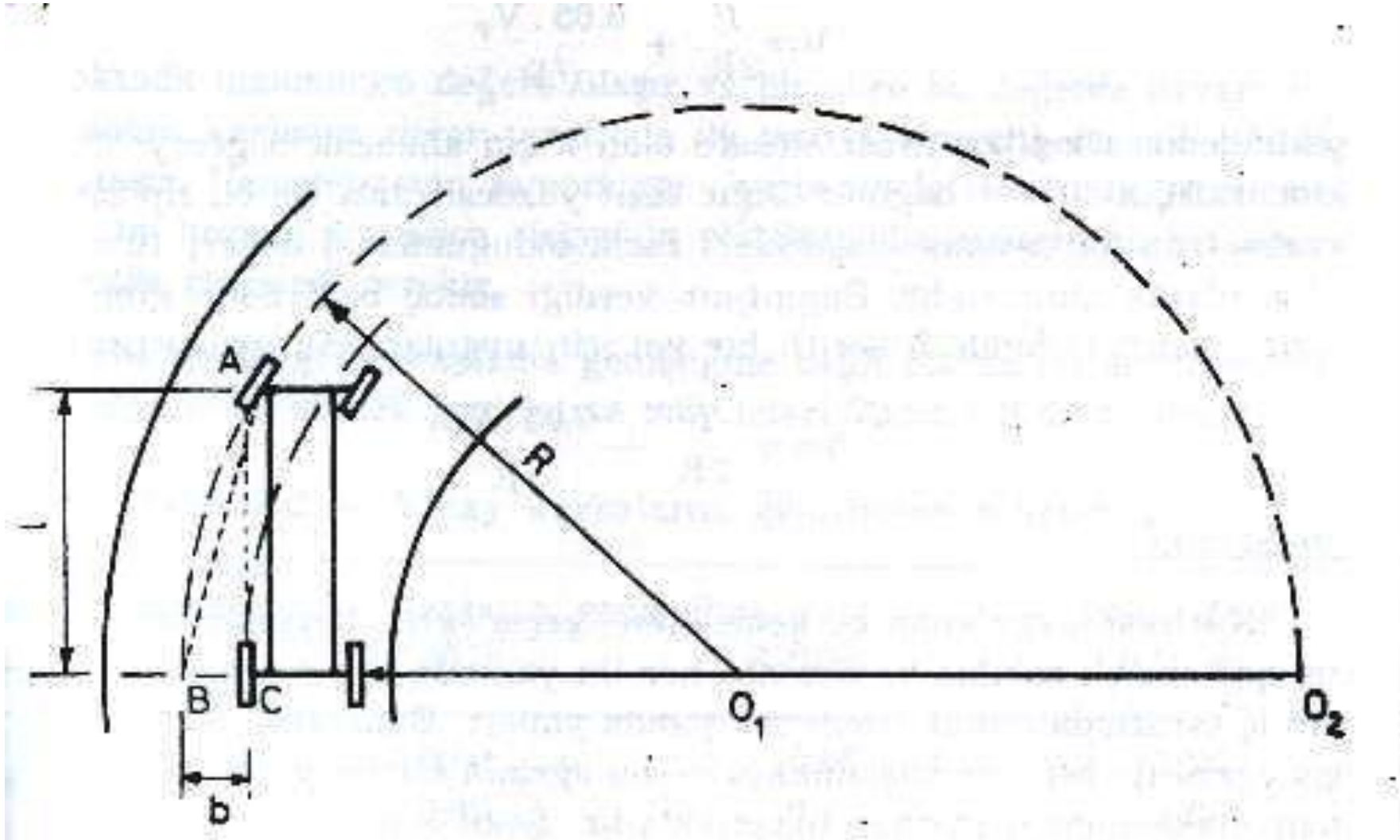


ADNAN MENDERES UNIVERSITY FACULTY OF ENGINEERING CIVIL ENGINEERING DEPARTMENT	
Superelevation Detail andDiagram	
Student Name	Student ID
Date	Scale:1/10 - 1/ 500

Travelled Way Widening on Horizontal Curves

Widening is needed on certain curves for one of the following reasons:

- **The design vehicle occupies a greater width because the rear wheels generally track inside front wheels (offtracking) in negotiating curves**
- **Drivers experience difficulty in steering their vehicles in the center of the lane.**


A
R
B
C
O_1
O_2
b

$$b = \frac{l^2}{2R}$$

l = The distance between the front and rear axles of vehicle (or vehicle length)(m)

R = vehicle's front outer wheel drawn the radius (the radius of the axis of the curve) (m)

B = the amount of widening for a single lane (m)

Above equation can be valid on low speeds. But, the higher the speed, the opposite lane violations may increase. Therefore, widening on high speed section is more calculated.

Widening on a single lane road →

$$b = \frac{l^2}{2R} + \frac{0{,}05.V_p}{\sqrt{R}}$$

$$b = n\frac{l^2}{2R} + \frac{0{,}05.V_p}{\sqrt{R}}$$

V_p= design speed (km/h); b= widening on n lanes roads

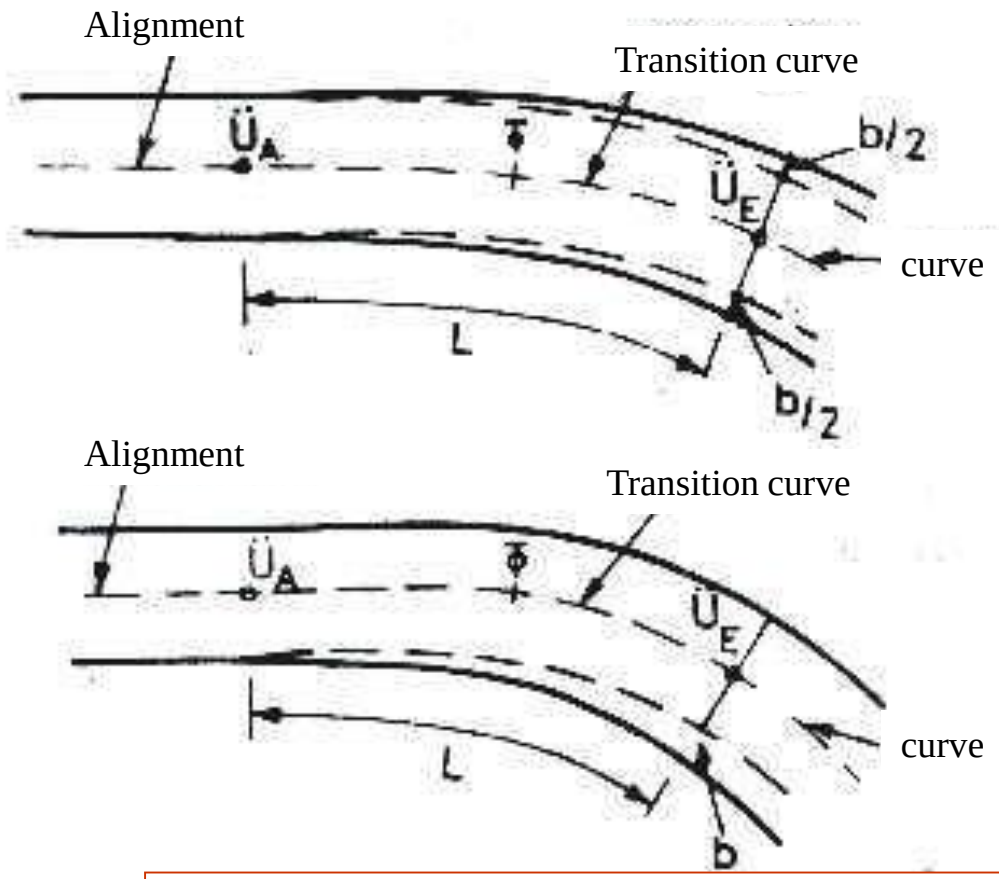
Application of Widening

Widening increases linearly along transition curve. Widening reachs max. value at the end of transition curve.

Widening is made two patterns;

- Center line is constant, **equal widening on inside and outside edges**
- After inside edge is widened, center line is shifted.

Application of widening in transition curve case

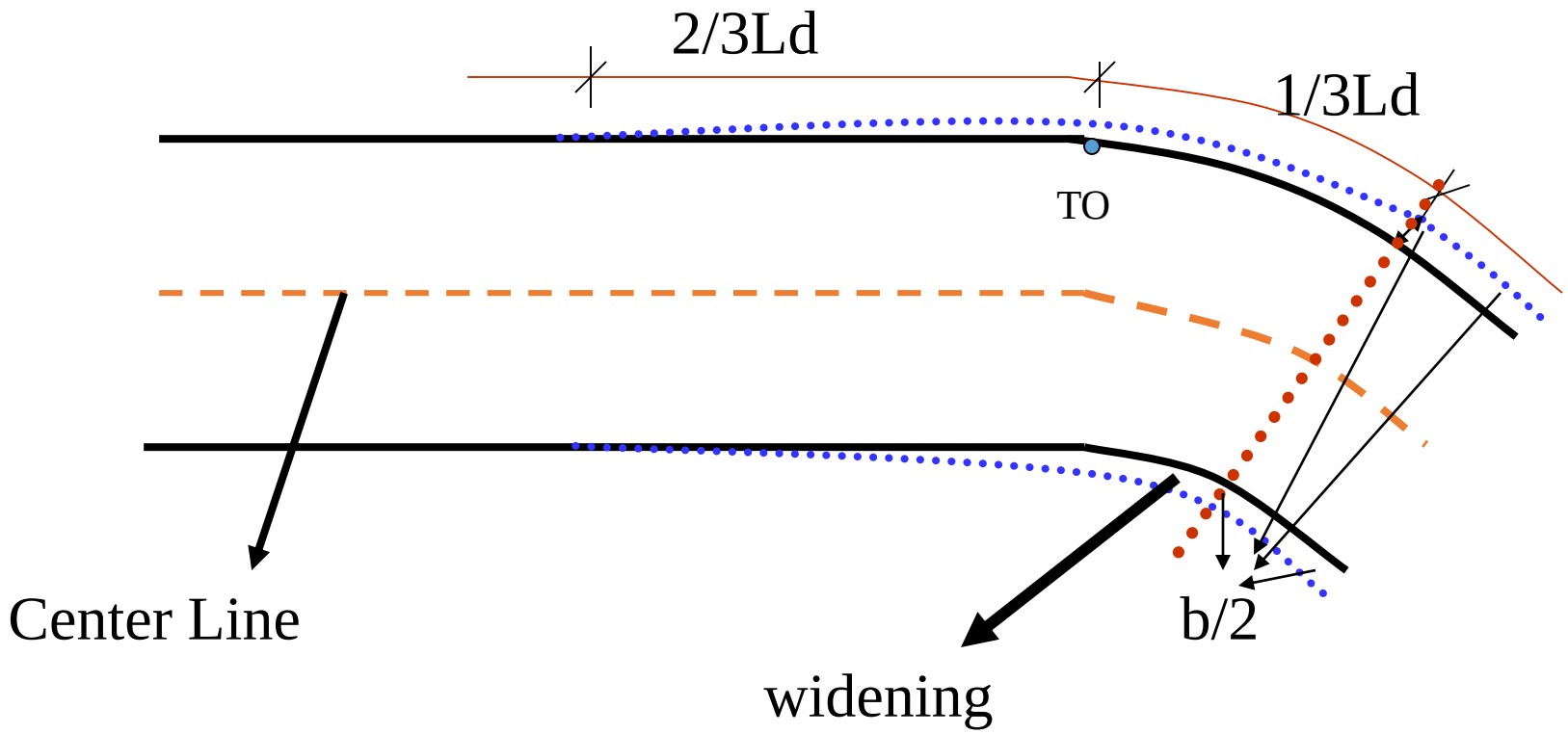


Widening on horizontal curves

Application of widening without transition curve case

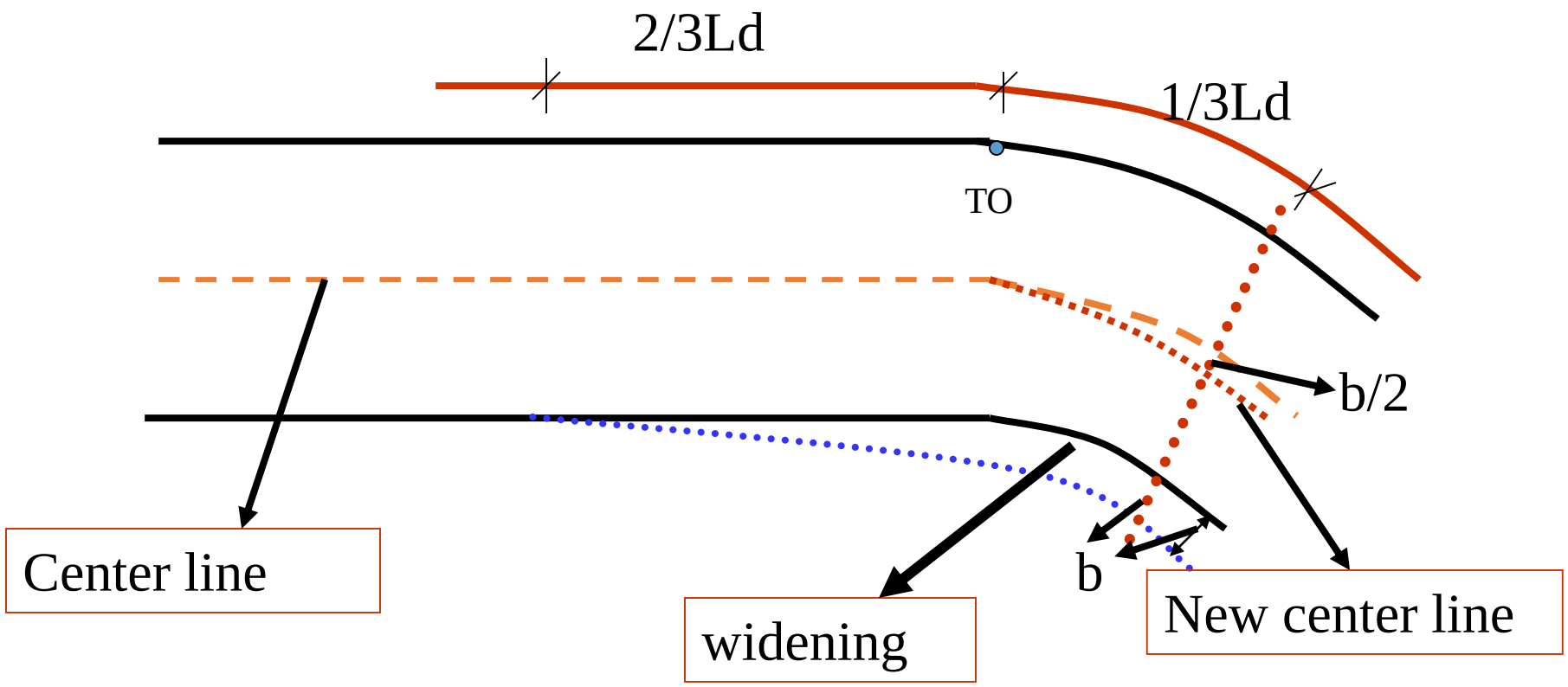
(center line is constant, equal widening on inside and outside edges)

Widening begins at the start point of length of raccordement.
Widening reachs max. value from point of TO to distance of 1/3Ld .

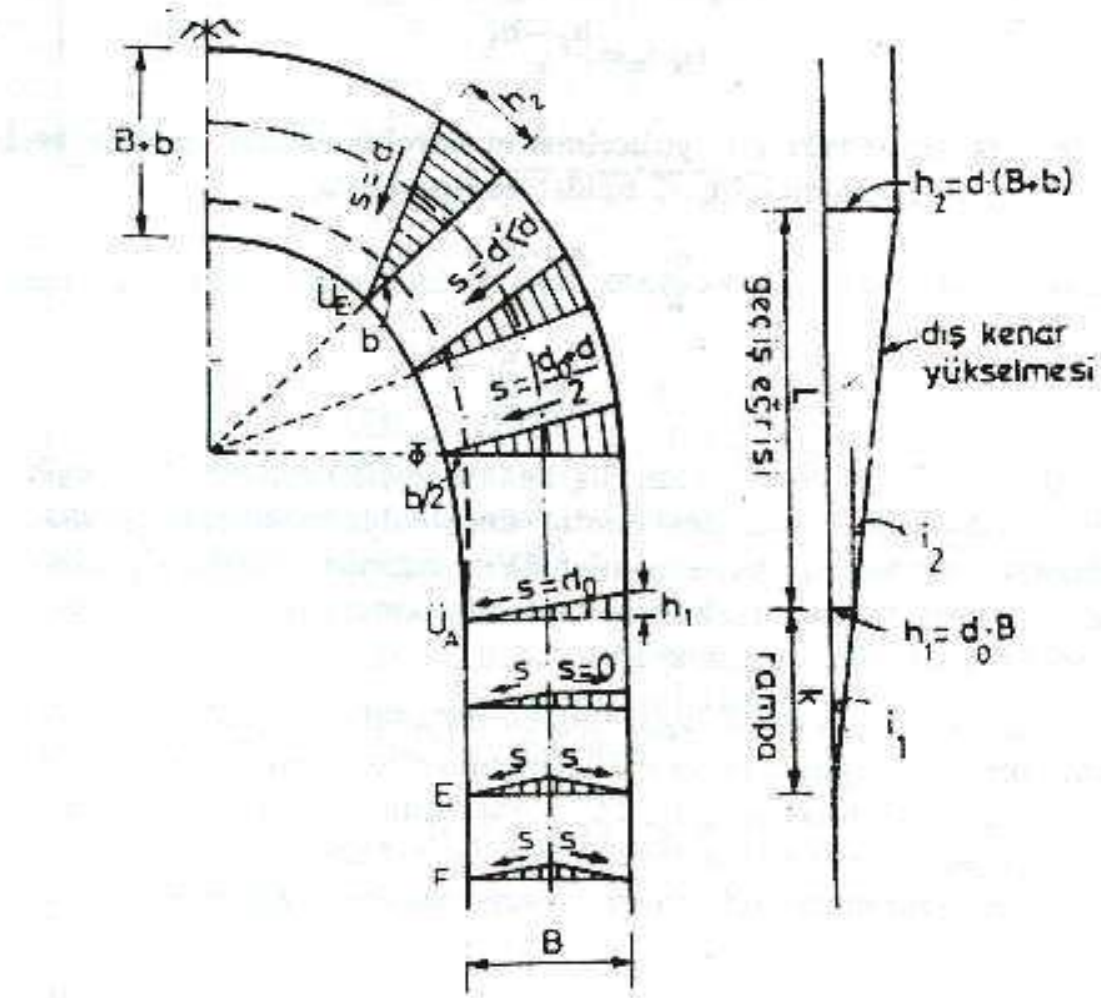


Widening without transition curve

(widening on inside edge)



Application of superelevation on horizontal curve which has transition curve and widening



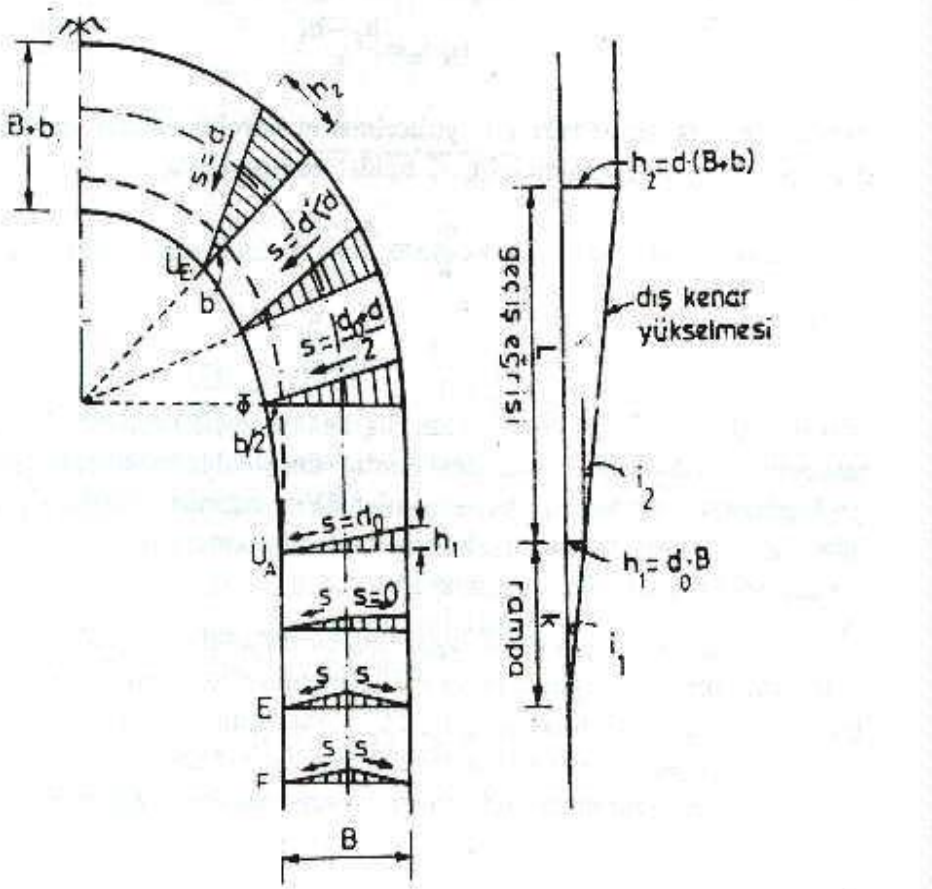
Application of Superelevation with Transition Curve

- When end of length of run out (the start point of run off Ü$_A$), inside and outside edge's crown should be equal.
- From point of Ü$_A$ (the start point of run off) to point of Ü$_E$ (the end point of run off) superelevation increases and reaches its max. value.
- Superelevation goes on its max. Value in the curve.
- At the exit curve, above operations continue adversely and reaches alignment

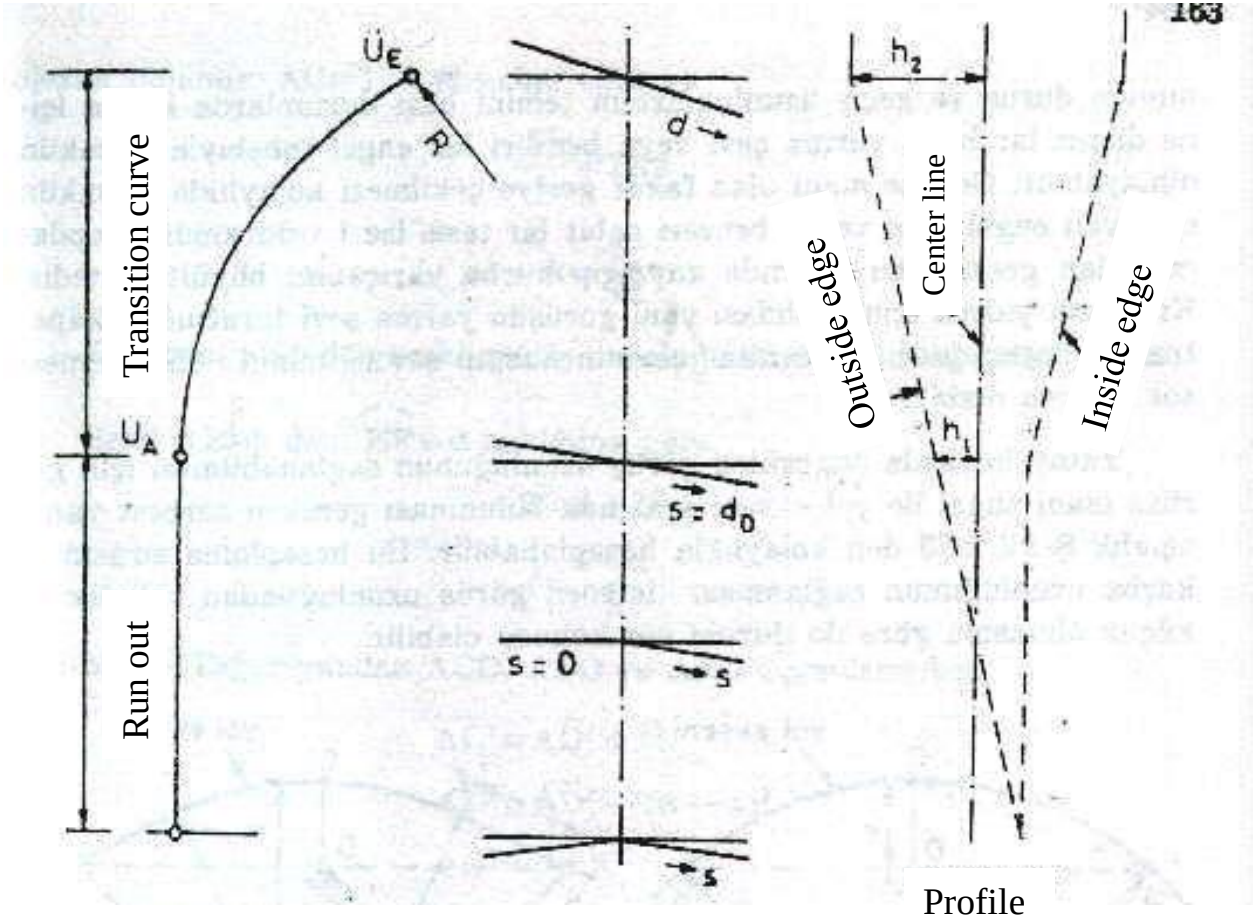
Lenght of Run out (inside edge profile is determined)

$$tgi_1 = \frac{h_1}{k} \qquad tgi_2 = \frac{h_2 - h_1}{L} \qquad (tgi_1 = tgi_2)$$

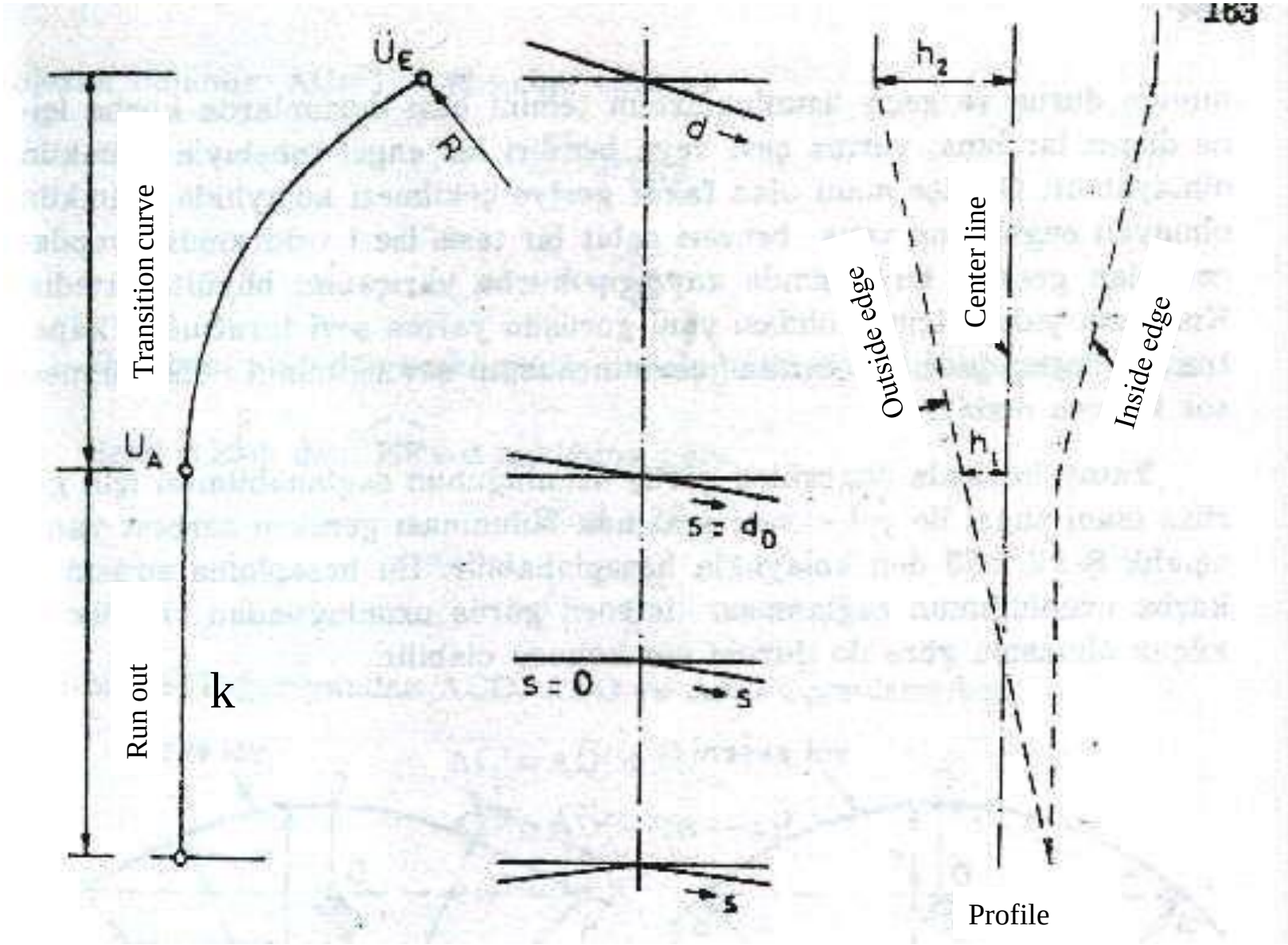
$$\frac{h_1}{k} = \frac{h_2 - h_1}{L} \Rightarrow k = \frac{L.h_1}{h_2 - h_1}$$



Lenght of Run out (center line profile is determined)

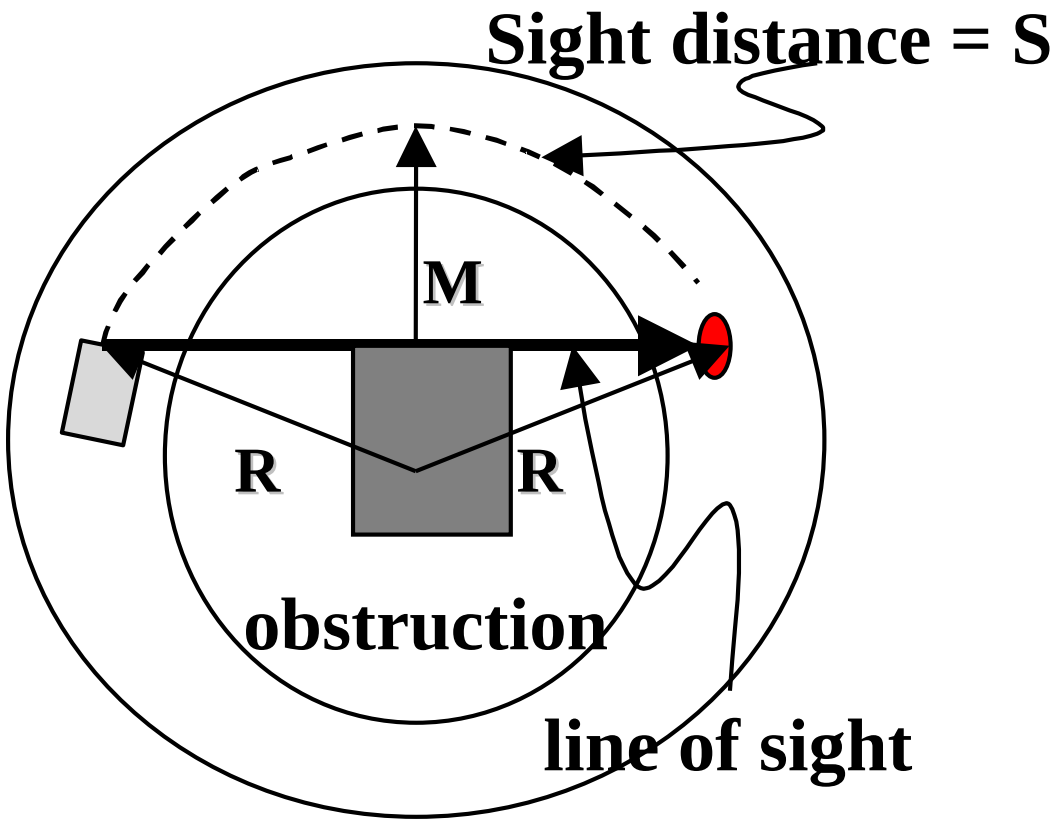


$$k = \frac{2h_1 . L}{h_2 - h_1}$$


Transition curve
Run out
k
U_E
U_A
R
d
s = d_0
s = 0
s
s
h_2
h_1
Outside edge
Center line
Inside edge

Profile

Sight Distance on Horizontal Curves

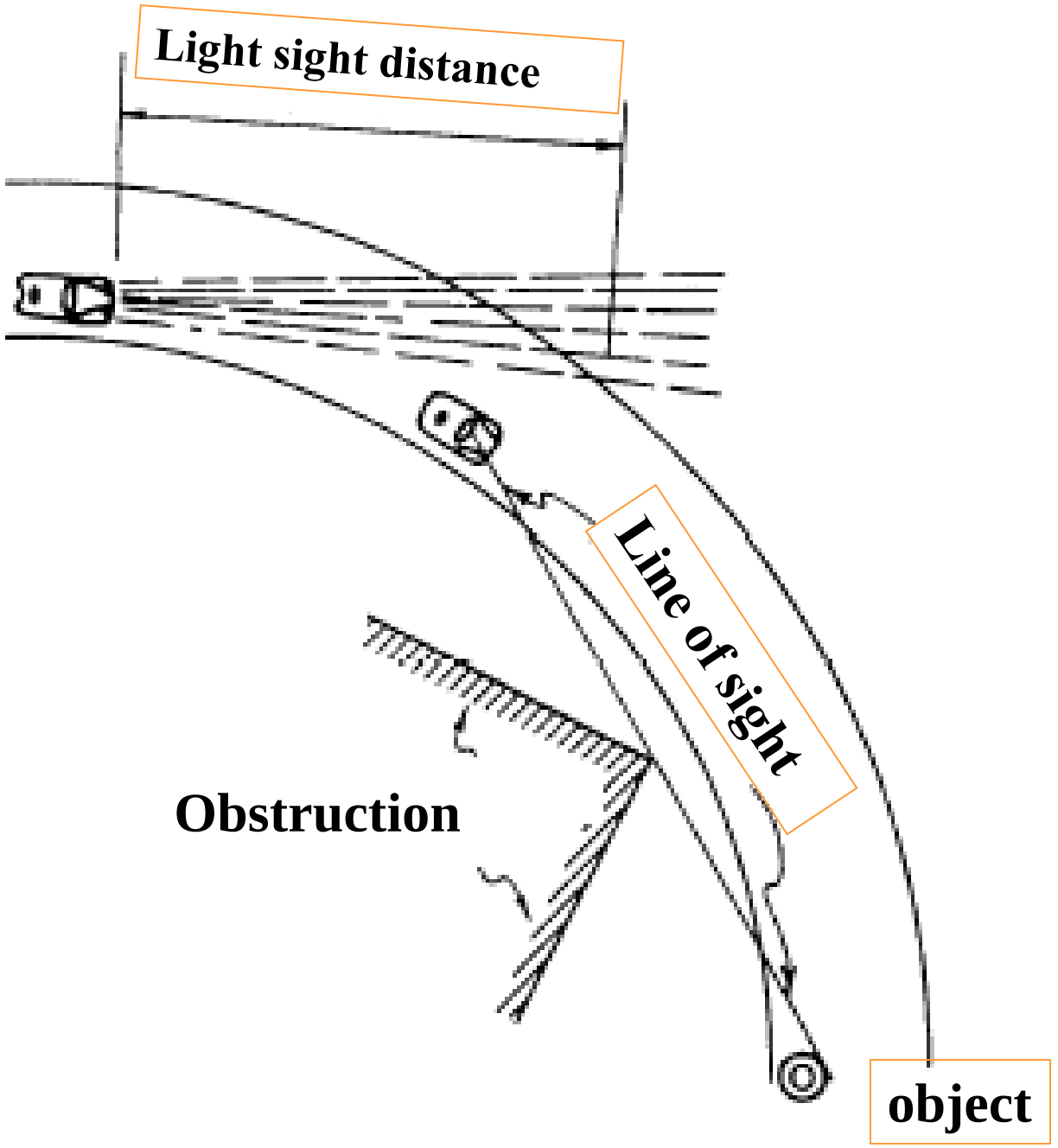


Sight Distance on Horizontal Curves

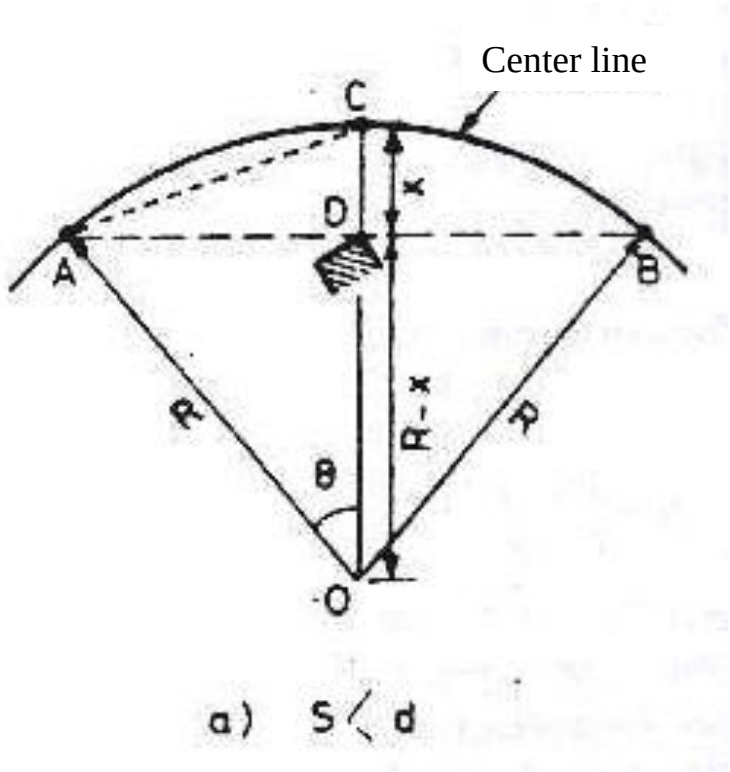
Sight distance is decreased by obstructions, cut slope in horizontal curves etc.

Solution is;

- **if possible, removing that obstruction,**
- **if not, radius should be increased or location shoul be changed. If there is cut slope, cut slope should be trimmed.**


Light sight distance
Line of sight
Obstruction
object

S<d



$AC^2 = AD^2 + x^2$

$AD^2 = R^2 - (R - x)^2$

$AC^2 = 2Rx$

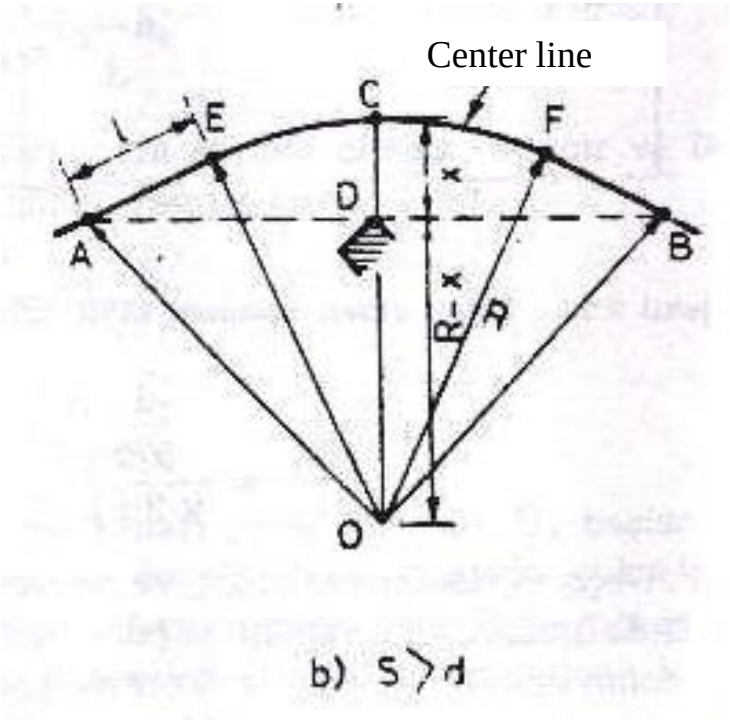
Accepting $AC = \frac{1}{2} S$

$S^2/4 = 2Rx$

$X = S^2/8R$

d= total length of circular curve from TO toTF measured along its arc.

S<d



EF= d= total length of circular curve from TO toTF measured along its arc

S= d+ 2*l*

***l*= (S – d)/2**

On the other hand, from ACD, ADO and EAO triangles

$$AC^2 = AD^2 + x^2 \quad AD^2 = AO^2 - (R - x)^2 \quad AO^2 = l^2 + R^2$$

accepting AC= 1/2S

x= d.(2S – d)/8R (shift distance)